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ECONOMIC GROWTH RISKS BASED ON DYNAMIC MODELS

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Poznyak S. P., Kolyada Yu. V. Economic Growth Risks Based on Dynamic Models

The article examines the theoretical foundations and practical approaches to defining the stationary state of the economy, analyzing the trajectory of economic growth, and assessing the accompanying economic risks. In conditions of global instability, financial shocks, and systematic internal imbalances, the substantiation of methods for assessing equilibrium states and identifying factors that may cause deviations of the actual development trajectory from the theoretically substantiated one becomes particularly significant. The article reveals the essence of the stationary state as the ultimate aim of the transitional process (dynamic trajectory), reflecting the balance of all macroeconomic parameters. It is to this state that the economic system aspires under the influence of fundamental parameters – the savings rate, population growth rates, capital depreciation, and levels of technological development, labor productivity, and investment activity. The study utilizes orthodox (one-dimensional) models of economic dynamics, specifically the Solow model, the Ramsey – Cass – Koopmans model, the Mankiw – Romer – Weil model, as well as the proposed authors' (multidimensional) system of equations that reflects intersectoral relationships within the country's economy. A new approach to the quantitative assessment of economic risk is proposed based on the interval of estimates of capital intensity over time, which allows for tracking the degree of deviation from the stationary state and forming an understanding of the economy's resilience to negative internal and external factors. The mechanism for identifying a numerical measure of risk based on the behavior of the trajectory of economic development is also substantiated, and the consequences of a high level of risk for the investment climate, dynamics of productivity, employment, financial stability, and the overall socioeconomic security of the country are discussed. The results of the research can be used to improve the quality of macroeconomic forecasts, strategic development planning, and the formulation of effective economic policy in a turbulent external environment.

Keywords: *steady state of the economy, trajectory of economic development, economic risk, economic growth, models of economic growth.*
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Позняк С. П., Коляда Ю. В. Ризики економічного зростання на підставі динамічних моделей

У статті досліджуються теоретичні засади та практичні підходи до визначення стаціонарного стану економіки, аналізу траєкторії економічного зростання та оцінки супутнього економічного ризику. В умовах глобальної нестабільності, фінансових потрясінь та систематичних внутрішніх дисбалансів особливого значення набуває обґрунтування методів оцінювання рівноважного стану та виявлення факторів, що можуть зумовлювати відхилення фактичної траєкторії розвитку від теоретично обґрунтованої. У роботі розкрито сутність стаціонарного стану як кінцевої мети перехідного процесу (динамічної траєкторії), що відображає збалансованість усіх макроекономічних параметрів. Саме до неї прагне економічна система під впливом фундаментальних параметрів – норми заощадження, темпів приросту населення, амортизації капіталу та рівня технологічного розвитку, продуктивності праці й інвестиційної активності. У дослідженні використовуються ортодоксальні (одновимірні) моделі економічної динаміки, зокрема модель Солоу, модель Рамсея – Касса – Купманса, модель Менк'ю – Ромера – Вейла, а також запропонована авторська (багатовимірна) система рівнянь, що відображає міжсекторальні взаємозв'язки в межах економіки країни. Запропоновано новий підхід до кількісної оцінки економічного ризику на основі інтервалу оцінок капіталомісткості в часі, що дозволяє простежити ступінь відхилення від стаціонарного стану та сформувати уявлення про стійкість економіки до негативних факторів внутрішнього та зовнішнього характеру. Також обґрунтовано механізм ідентифікації числової міри ризику за поведінкою траєкторії економічного розвитку, зазначено наслідки високого рівня ризику для інвестиційного клімату, динаміки продуктивності, зайнятості, фінансової стабільності та загальної соціально-економічної безпеки держави. Результати дослідження можуть бути використані для підвищення якості макроекономічних прогнозів, стратегічного планування розвитку та розробки ефективної економічної політики в умовах турбулентного зовнішнього середовища.

Ключові слова: *стаціонарний стан економіки, траєкторія економічного розвитку, економічний ризик, економічне зростання, моделі економічного зростання.*
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The etymology of economic risk is described in detail in [1]. In today's environment (global instability, growing economic challenges, etc.), preventive assessment of sustainable economic growth is of particular relevance. The ability of the economy to maintain the optimal level of production and adapt to internal and external shocks determines both the welfare of society and the prospects (medium or long-term) of the country's competitiveness. The nonlinear dynamics of economic evolution is accompanied by various fluctuations, structural changes, and various risks, which can significantly slow down the transition to a stationary state or significantly deviate from the equilibrium point.

The study of the equilibrium state as a special case of the dynamic trajectory of the economic system and its risks is extremely important for the development of effective macroeconomic policies aimed at ensuring the sustainability and predictability of economic growth in the medium and long term. This is especially true for emerging economies, as their vulnerability to external shocks, global market fluctuations, financial instability, and internal structural imbalances can quickly lead to a loss of macroeconomic balance and an aggravation of socioeconomic problems.

Understanding the nature of the steady state allows us to identify the fundamental limitations of the economic system, determine its potential for sustainable growth, and assess whether the current development trajectory is in line with equilibrium parameters. An analysis of the factors that deviate the economy from steady state, as well as the risks that exacerbate fluctuations in macroeconomic indicators, provides the basis for developing effective mechanisms to prevent crises and mitigate their consequences.

The article analyzes the key theoretical and methodological approaches to determining the steady state of the economy, reveals the mechanisms of influence of both internal and external factors on the trajectory of economic growth, and provides a comprehensive assessment of the risks accompanying these processes. The results obtained are of great practical importance for improving forecasting models and increasing the accuracy of economic forecasts.

Analysis of recent research and publications. Recent studies have increasingly explored the quantitative estimation of steady-state growth using advanced econometric techniques. For instance, Kohlscheen and Nakajima [2] employ a time-varying parameter structural VAR model to estimate real-time benchmark growth rates for major economies, demonstrating a decline in the U.S. steady-state GDP growth from above 3 % in the 1990s to around 2.4% by 2019. This objective benchmark highlights the presence of persistent slack and potential risk emerging from deviations between observed output and the estimated equilibrium.

In the field of multidimensional risk quantification, Matutinović et al. [3] offer a simulation model capturing theoretical conditions for a global steady-state economy, focusing on how economic trajectories respond under systemic constraints. Complementarily, the IMF's work on model-based risk assessment [4] constructs predictive distributions of macro variables using non-linear structural models, addressing downside tail risks and skewness in growth forecasts. These approaches align closely with our proposed framework for tracking deviations via capital intensity intervals and quantifying economic resilience.

Other recent contributions include empirical analyses of sustainable growth drivers. Wang et al. [5] study developed economies and find that renewable energy consumption and scientific progress significantly promote sustainable GDP expansion, with elasticity estimates of 0.12% and 0.35%, respectively. Additionally, Klimek, Poledna, and Thurner [6] introduce input-output susceptibility as a nonequilibrium resilience measure, showing that sector-specific vulnerabilities improve forecasts of economic recovery post-shocks. These studies reinforce the relevance of multidimensional indicators – energy, technology, and structural resilience – in understanding deviations from the steady state and their policy implications.

Highlighting previously unresolved parts of the overall problem. Despite extensive research on economic growth modelling, several aspects remain insufficiently addressed. First, the practical integration of the concept of a steady state with real-world development trajectories is often limited to purely theoretical frameworks without a mechanism for empirical identification of deviations. Classical one-dimensional models provide an abstract understanding of long-term equilibrium but do not reveal the multidimensional nature of economic dynamics under conditions of global instability and persistent internal imbalances.

Furthermore, the existing literature rarely offers a robust quantitative measure of economic risk associated with deviations from the steady state. While many studies recognize the presence of risk factors, they often lack an approach to evaluate the degree of risk based on the dynamic behavior of the economy's capital intensity and other fundamental parameters over time. This gap restricts the practical applicability of growth models for macroeconomic forecasting and risk management.

Lastly, the relationship between the identified level of economic risk and its implications for the investment climate, productivity growth, and socio-economic stability remains underexplored. Prior research does not sufficiently link deviations in the development trajectory with concrete outcomes for economic resilience. This study addresses these unresolved aspects by proposing an approach to risk quantification rooted in the trajectory's behavior, thus bridging theoretical constructs with practical economic analysis.

The *aim* is to develop and substantiate a methodological framework for assessing the steady state of the economy, analyzing the trajectory of economic growth, and quantifying the associated economic risk. By integrating orthodox growth models with an original multidimensional system of equations, the study aims to identify the fundamental parameters that determine the long-term equilibrium and to propose a practical approach for measuring deviations from it.

A steady state of economic development is an equilibrium state of the economic system in which the main macroeconomic indicators (output, capital and consumption per capita, etc.) remain stable over time, i. e., within fixed limits, even when the overall level of production grows in line with the rate of population growth or technological progress. In other words, in a steady state, the economy moves along a steady development trajectory, where the rate of capital accumulation fully compensates for its depreciation and provides the necessary level of resources for new generations.

Achieving steady state depends on many factors. The most important among them are the level of savings and investment, which determines the opportunities for capital accumulation, the rate of population growth, which affects the distribution of goods and resources produced per capita, and the rate of technological progress, which increases labor productivity and resource efficiency. The depreciation rate, which characterizes the rate of depreciation of fixed assets, as well as institutional and political conditions that create a favorable environment for stable economic development and minimization of economic risks, play an important role.

A number of classical and modern economic models are used to estimate the steady state. The most well-known is the Solow – Swan model, which explains how savings, population growth, and technological progress interact to determine the equilibrium level of capital and output per capita. The Mankiw – Romer – Weil model complements this concept by introducing human capital as a separate factor of production, which allows for better accounting for the qualitative characteristics of the labor force and explaining differences in income levels between different countries. At the same time, the Ramsey – Cass – Koopmans model offers an endogenous approach to modeling economic growth by considering household decisions on the intertemporal distribution of consumption and savings, which allows for a deeper understanding of the mechanisms of reaching a steady state. In addition to these models, other modifications are used that take into account the impact of economic openness, the role of the public sector, structural changes and external shocks, which allows for flexible assessment of the equilibrium state and development of sound economic policies to ensure sustainable economic development.

The trajectory of economic growth is a sequence of changes in key macroeconomic indicators over time, which reflects the movement of the economy from an initial state to a final state – a stationary or equilibrium state. In other words, it is a dynamic path by which an economic system gradually approaches its steady state, determined by such fundamental parameters as the savings rate, population growth rate, level of technological progress, and capital depreciation. The trajectory of economic growth shows how production, capital, and consumption per capita change in the process of transition from one state to another, taking into account external influences and internal structural changes.

In the short and medium term, the trajectory of an economy can deviate significantly from its steady state due to cyclical fluctuations, supply or demand shocks, changes in investment activity, or the impact of economic policy. However, in the long run, the economy usually tends to return to an equilibrium trajectory if there are no significant disturbances in the fundamentals.

Economic growth models allow us to theoretically describe not only the steady state itself, but also the entire process of achieving it, determining the shape of the trajectory of convergence or divergence from the equilibrium level. In particular, models [7–16] demonstrate how an economy starting with a low level of capital grows rapidly due to high marginal productivity of capital, and as it approaches steady state, the growth rate gradually slows down.

Economic growth risk is a numerical measure of the probability that the actual trajectory of economic development will deviate from the planned or forecasted level of growth due to the impact of unfavorable internal or external factors. In other words, growth risk reflects the degree of uncertainty regarding the achievement of stable economic development rates and the maintenance of the economy on a trajectory leading to a stationary or equilibrium state.

The level of economic growth risk is affected by both external and internal factors. External factors include global economic shocks, fluctuations in raw material and energy prices, changes in global market conditions, international financial instability, geopolitical tensions and trade conflicts. Domestic factors include macroeconomic imbalances, inefficient production structure, low level of innovation and technological development, unstable political situation, weak institutions and corruption. Also important are factors related to demographic dynamics, the quality of the labor force and the availability of capital for investment.

A high degree (level) of economic growth risk has a number of negative consequences for the economy, which manifest themselves in both the short and long term. First and foremost, it leads to increased volatility of macroeconomic indicators: GDP growth rates become unstable, the cyclicality of development intensifies, and the likelihood of sharp recessions and protracted crises increases. This has a negative impact on the investment climate, as increased uncertainty reduces the propensity of businesses and households to invest and plan for the long term. As a result, the volume of capital investments decreases, the renewal of production capacities and the introduction of innovations slowdown, which in turn worsens labor productivity and economic competitiveness.

In addition, the high level of risk is accompanied by rising costs of financing the economy: banks and investors build risk premiums into interest rates and project profitability requirements. This complicates access to credit and investment resources, especially for small and medium-sized businesses. In the social dimension, unstable economic growth can lead to lower incomes, higher unemployment, and a deterioration in the quality of life, which increases social tensions and distrust in the government.

A high level of risk can be identified by analyzing the trajectory of economic development. The main signs are frequent and sharp fluctuations in production growth, the absence of a clear tendency to converge with the equilibrium or steady state, constant deviations of actual indicators from the forecasted levels, and a tendency to reduce the stability of investment and consumption. If the trajectory of the economy is characterized by repeated boom and bust cycles with no signs of long-term leveling off, this indicates the presence of high internal or external risks.

Research methodology: Using one-dimensional dynamic models. To model economic growth, we will consistently consider the Solow – Swan [7–9], Ramsey – Cass – Koopmans [10–12], Mankiw – Romer – Weil [13], and the author's model described in [14–16].

Solow's nonlinear model of economic growth was developed in 1957 [7–9]. It was based on the Cobb – Douglas production function, where the economy was described by two factors: capital and labor. The formula of the equation is as follows:

$$Y = AK^{\alpha}L^{\beta}, \qquad (1)$$

where α and β are the elasticities of change of capital and labor, respectively, and the coefficient A responsible for technical progress. Additional assumptions of the model are as follows: capital intensity $k = K / L$ does not remain constant, as assumed in Keynesian approaches, but changes depending on macroeconomic conditions in the country; prices, goods, services, and resources are formed by the market mechanism; the growth rate of labor resources corresponds to the average growth rate of the population, but changes in wages are not taken into account in the dynamics. It is also assumed that at the initial stage there is no population growth and no technological progress. The parameters of the savings rate, depreciation, technical progress, capital and labor elasticities, and population growth rates are assumed to be constant, although in real conditions they change over time [7–9].

Finally, Solow's equation takes the form:

$$k^{\bullet} = sAk^{\alpha} - (d+n)k, \;\; k_0 = k(t_o), \qquad (2)$$

where variable $k = k(t)$ corresponds to the capital intensity; $k^{\bullet} = \dfrac{dk}{dt}$ – its first derivative; coefficient s – capital accumulation rate; constants A and α are the Cobb – Douglas functions, accordingly A reflects indirect costs, and the value of α is the elasticity; the coefficient d – the degree of capital disposal; n – average growth rate of the employed population.

In the Solow model, steady state refers to an equilibrium in which the amount of capital per capita remains constant over time, as capital gains from investment are exactly offset by depreciation and the needs arising from population growth. In this state, the rate of economic growth is determined only by the rate of population growth and the rate of exogenous technical progress, while at the level of one worker, output and capital are stabilized at a constant level. The steady state in the Solow model is determined by fundamental parameters: the savings rate, the population growth rate, the depreciation rate, and the level of technological development. It is this state that the economy tends to reach in the long run, and deviations from it are gradually smoothed out by the mechanism of equalization through changes in the marginal productivity of capital. The steady state corresponds to the expression:

$$k^* = \left[\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}, \qquad (3)$$

where s – the higher the share of savings, the more resources for investment – more sustainable capital;

A – level of technology: the more efficient the production, the more can be produced per unit of capital; $d+n$ – "burden" that needs to be covered by investments. Thus, the fraction $\frac{sA}{d+n}$ is the net investment rate per effective employee, which covers losses and gains.

Note: The size of the fraction increases as the numerator increases and decreases as the denominator increases. This is the balance between how much the economy invests (through savings and technology) and how much is needed to compensate for depreciation, population growth, and technological progress. The higher the savings rate or technology, the higher the fixed capital stock. The higher the depreciation, population growth, or rate of technological progress, the lower the sustainable capital.

By integrating the differential equation (2), we write its analytical solution:

$$k(t)=\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}. \quad \mathbf{(4)}$$

Equation (4) can also be rewritten as:

$$k(t)=\left[\left(k_0^{1-\alpha}-k^{*1-\alpha}\right)e^{-(d+n)t}+k^{*1-\alpha}\right]^{\frac{1}{1-\alpha}}. \quad \mathbf{(5)}$$

Or:

$$k(t)=\left[\left(k_0^{1-\alpha}e^{-(d+n)t}+k^{*1-\alpha}\left(1-e^{-(d+n)t}\right)\right)\right]^{\frac{1}{1-\alpha}}, \quad \mathbf{(6)}$$

where $k_0^{1-\alpha}$ – is responsible for the initial level of capital intensity, which is adjusted for elasticity; $k^{*1-\alpha}$ – is responsible for the steady-state level of capital intensity, which is also adjusted for elasticity $\left(k_0^{1-\alpha}-k^{*1-\alpha}\right)$ – means the initial deviation from the steady state; $e^{-(d+n)t}$ – speed of convergence to the steady state.

The calculation of economic risk based on capital intensity involves the use of an interval of estimates of this indicator for the current period of time. To do this, you need to determine the expected upper limit $k(t)^i_{\max}$ and the lower limit $k(t)^i_{\min}$, reflecting, respectively, the most optimistic and pessimistic scenarios of the capital intensity level. The economic risk itself in this approach is estimated as the relative spread of this interval relative to its middle and is determined by the formula:

$$R^i=\frac{k(t)^i_{\max}+k(t)^i_{\min}}{(k(t)^i_{\max}-k(t)^i_{\min})/2}. \quad (7)$$

Factor (7) demonstrates the degree of uncertainty in the value of capital intensity at a given point in time. The wider the gap between the upper and lower estimates, the higher the risk of economic development, as this indicates greater variability or unpredictability of the resource provision per employee. This method allows quantifying risk as a function of the spread of forecasts of the key parameter of the growth model and is used to assess the reliability of economic forecasts and develop policies taking into account possible fluctuations.

The expected upper limit can be calculated as:

$$k(t)_{\max}=\left[\left(k_0^{1-\alpha}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}. \quad \mathbf{(8)}$$

In equation (8), the first term reflects how the initial level of capital intensity decreases over time due to capital depreciation and population growth. The longer the time period and the higher d and n, the faster this effect reduces the contribution of initial capital. The second term reflects the equilibrium contribution of savings and productivity to the maintenance of capital per worker. This is the "steady-state" part that capital intensity tends to reach in the long run. It shows the balance between investment financed by savings and capital losses due to depreciation and population growth. Thus, formula (8) shows the maximum possible level of capital intensity per worker under ideal conditions – that is, when the economy fully realizes its savings potential and does not face any negative deviations or shocks.

The expected lower bound is given by the equation:

$$k(t)_{\min}=\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha}}. \quad \mathbf{(9)}$$

Formula (9) describes a scenario where the economy develops under less favorable conditions compared to the baseline or "optimistic" scenario. Unlike the upper bound formula, where the second term with savings is added and "pulls" the capital intensity upward, it is subtracted here, which shows that the actual level of investment or its efficiency may be lower than the estimated maximum. In particular, the expression in brackets $\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)$ means that the initial level of capital intensity is partially "adjusted downward" by the equilibrium contribution of savings. This corresponds economically to a situation where a part of potential investment resources is not realized for various reasons, such as low financial market efficiency, losses due to institutional problems, or external shocks. Multiplication by an exponent $e^{-(d+n)t}$ shows how this reduced capital stock is further reduced over time by depreciation and population growth. Thus, the formula describes the trajectory of capital intensity under the minimum possible scenario, when the economy is not able to fully compensate for capital depreciation and population growth through its own savings.

Taking into account formulas (8) and (9), the risk of economic growth for the Solow model can be determined by the formula:

$$R=\frac{\left[\left(k_0^{1-\alpha}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}+\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha}}}{\left(\left[\left(k_0^{1-\alpha}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}-\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha}}\right)/2}. \quad (10)$$

But given the inequality $\frac{a+b}{2}\geq\sqrt{ab}$ for positive numbers *a, b* there will be an approximate assessment of the degree of risk:

$$R\leq\sqrt{\frac{\left[\left(k_0^{1-\alpha}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{1-\alpha}}}{\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha}}}}. \quad (11)$$

Or:

$$R\leq\sqrt{\frac{\left[\left(k_0^{1-\alpha}\right)e^{-(d+n)t}+\frac{sA}{d+n}\right]^{\frac{1}{2(1-\alpha)}}}{\left[\left(k_0^{1-\alpha}-\frac{sA}{d+n}\right)e^{-(d+n)t}\right]}}. \quad (12)$$

The Mankiw – Romer – Weil model [13] is essentially a modification of the Solow model with the addition of human capital (*H*). Thus, the production function is written as follows:

$$Y=AK^{\alpha}H^{\beta}L^{1-\alpha-\beta}, \quad (13)$$

and the dynamic model itself takes the form of a system of differential equations as:

$$\begin{aligned} k^{\bullet}&=s_kAk^{\alpha}h^{\beta}-(d+n)k,\ k_0=k(t_o),\\ h^{\bullet}&=s_hAk^{\alpha}h^{\beta}-(d+n)h,\ h_o=h(t_o), \end{aligned} \quad (14)$$

where s_k – rate of accumulation of physical capital, a s_h – rate of human capital accumulation, *h* – human capital per unit of labor.

The steady state for the capital intensity of production in the model can be found as:

$$k^*=\left[\left(\frac{s_k}{s_h}\right)^{\beta}\frac{s_kA}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}, \quad (15)$$

and for human capital stocks per unit of labor:

$$h^*=\left[\left(\frac{s_h}{s_k}\right)^{\alpha}\frac{s_kA}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}. \quad (16)$$

Integrating system (14), we can obtain analytical solutions for *k*(*t*) and *h*(*t*), respectively:

$$k(t)=\left[\left(k_0^{1-\alpha-\beta}-\frac{(s_k+s_h)A}{d+n}\right)+\frac{(s_k+s_h)A}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}; \quad (17)$$

$$h(t)=\left[\left(h_0^{1-\alpha-\beta}-\frac{(s_k+s_h)A}{d+n}\right)e^{-(d+n)t}+\frac{(s_k+s_h)A}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}. \quad (18)$$

Further calculations are shown in *Tbl. 1*.

Then the approximate risk will be calculated using the formulas:

$$R_k\leq\left[\frac{\left(k_0^{1-\alpha-\beta}\right)e^{-(d+n)t}+\frac{(s_k+s_h)A}{d+n}}{\left(k_0^{1-\alpha-\beta}-\frac{s(s_k+s_h)A}{d+n}\right)e^{-(d+n)t}}\right]^{\frac{1}{2(1-\alpha-\beta)}}, \quad (19)$$

and:

$$R_h\leq\left[\frac{\left(h_0^{1-\alpha-\beta}\right)e^{-(d+n)t}+\frac{(s_k+s_h)A}{d+n}}{\left(h_0^{1-\alpha-\beta}-\frac{s(s_k+s_h)A}{d+n}\right)e^{-(d+n)t}}\right]^{\frac{1}{2(1-\alpha-\beta)}}, \quad (20)$$

for physical and human capital stocks per unit of labor, respectively.

The Ramsey – Cass – Koopmans model [10–12] is based on a similar production function as (1). Its basic formula can be expressed through the system of equations:

$$\begin{aligned} k^{\bullet}&=Ak^{\alpha}-c-(d+n)k,\ k_0=k(t_0),\\ c^{\bullet}&=\frac{1}{\theta}(\alpha Ak^{\alpha-1}-r-\rho)c,\ c_0=c(t_o), \end{aligned} \quad (21)$$

where θ – constant elasticity of substitution *r* – the rate of decrease in assets per unit of labor, ρ – coefficient of intertemporal preference of the consumer.

The steady state for capital intensity is equal to:

$$k^*=\left[\frac{\alpha A}{r+p}\right]^{\frac{1}{1-\alpha}}, \quad (22)$$

and for consumption per unit of labor:

$$c^*=A\left[\frac{\alpha A}{r+p}\right]^{\frac{\alpha}{1-\alpha}}-(n+d)\left[\frac{\alpha A}{r+p}\right]^{\frac{1}{1-\alpha}}. \quad (23)$$

Table 1

Index	Lower limit	Upper limit
k	$\left[\left(k_0^{1-\alpha-\beta}-\frac{(s_k+s_h)A}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha-\beta}}$	$\left[\left(k_0^{1-\alpha-\beta}\right)e^{-(d+n)t}+\frac{(s_k+s_h)A}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}$
h	$\left[\left(h_0^{1-\alpha-\beta}-\frac{(s_k+s_h)A}{d+n}\right)e^{-(d+n)t}\right]^{\frac{1}{1-\alpha-\beta}}$	$\left[\left(h_0^{1-\alpha-\beta}\right)e^{-(d+n)t}+\frac{(s_k+s_h)A}{d+n}\right]^{\frac{1}{1-\alpha-\beta}}$

An analytical solution of the Ramsey – Cass – Koopmans model in general (i. e., a complete closed solution) cannot be obtained due to its significant nonlinearity and interdependence of capital and consumption dynamics: the model equations form a nonlinear system of differential equations, where the change in consumption depends on the marginal productivity of capital, and the change in capital depends on the level of current consumption. Such a system does not have a closed solution in the form of elementary functions for arbitrary initial conditions and parameters, so in practice either stationary characteristics or numerical integration are used to analyze trajectories of convergence to equilibrium.

Formally, the solution can be written as a nonlinear integral:

$$\int\frac{dc}{c}=\int\frac{\frac{1}{\theta}(\alpha Ak^{\alpha-1}-r-\rho)c}{Ak^{\alpha}-c-(d+n)k}dk. \quad (24)$$

However, if we recall that c – is an exogenous variable and write it as $c_i k$, where $c_i = c/k$, then the steady state can be found by the formula:

$$k^*=\left[\frac{A}{d+n+c_i}\right]^{\frac{1}{1-\alpha}}. \quad (25)$$

Then the analytical solution will be:

$$k(t)=\left[\left(k_0^{1-\alpha}-\frac{A}{d+n+c_i}\right)e^{-(d+n+c_i)t}+\frac{A}{d+n+c_i}\right]^{\frac{1}{1-\alpha}}. \quad (26)$$

And the upper and lower bounds will be the expressions:

$$k(t)_{\max}=\left[\left(k_0^{1-\alpha}\right)e^{-(d+n+c_i)t}+\frac{A}{d+n+c_i}\right]^{\frac{1}{1-\alpha}}, \quad (27)$$

and

$$k(t)_{\min}=\left[\left(k_0^{1-\alpha}-\frac{A}{d+n+c_i}\right)e^{-(d+n+c_i)t}\right]^{\frac{1}{1-\alpha}}. \quad (28)$$

In accordance with formulas (27) and (28), an approximate numerical measure of economic risk is calculated by the formula:

$$R\le\left[\frac{\left(k_0^{1-\alpha}\right)e^{-(d+n+c_i)t}+\frac{A}{d+n+c_i}}{\left(k_0^{1-\alpha}-\frac{A}{d+n+c_i}\right)e^{-(d+n+c_i)t}}\right]^{\frac{1}{2(1-\alpha)}}. \quad (29)$$

Using the author's model of economic growth. For this model [14–16], the main factors of production are private capital K_{pr}, public capital K_{gov}, human capital (knowledge) H, labor L and the variable factor R. Variable factor R in a single-sector production model is responsible for the land factor N. A modified Cobb – Douglas function of the form:

$$Y_p=AK_{pr}^{\alpha}K_{gov}^{\beta}H^{\gamma}N^{\varphi}L^{1-\alpha-\beta-\gamma-\varphi}, \quad (30)$$

where α – is the coefficient of elasticity of private capital; β – public capital elasticity coefficient; γ – human capital elasticity coefficient; φ – elasticity of the variable factor, in this case, land.

Capital in the model is divided into private and public, which allows for a more accurate accounting of the differences in the functions and role of each sector in the process of economic growth. Investments are made through aggregate savings, which reflect the ability of the economy to effectively channel resources for development. Thus, capital dynamics can be described through three key indicators: private sector capital intensity, which determines the volume of private investment; public sector capital intensity, which characterizes investments in public infrastructure and public goods; and aggregate savings per unit of labor, which is the main source of investment in the economy. This approach allows for a deeper exploration of the interrelationships between private and public investment, as well as their joint impact on labor efficiency and long-term economic growth.

The innovation sector generates new knowledge by the production function:

$$\Delta H = BK_{rd}^{\upsilon} L_{rd}^{1-\upsilon}, \qquad \textbf{(31)}$$

where K_{rd} – capital raised in the innovation sector, L_{rd} – labor involved in the innovation sector, υ – elasticity of capital in the innovation sector. Total capital in the economy K_{full} can be found by the formula: $K_{full} = K_{rd} + K_{pr} + K_{gov}$, similar to labor: $L_{full} = L_{rd} + L$.

Full single-sector multivariate model in general form:

$$\begin{cases} k_{pr}^{\bullet} = i_{in} + i_f - (d_{pr} + n)k_{pr}, \\ k_{gov}^{\bullet} = g - (d_{gov} + n)k_{gov} + tx, \\ m^{\bullet} = sAk_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - (g + nm + i_{in} + i_{out}), \\ h^{\bullet} = Bk_{rd}^{\upsilon} l - nh, \end{cases} \qquad \textbf{(32)}$$

where k_{pr} – capital intensity of the private sector; d_{pr} – depreciation ratio of private capital; n – average growth rate of the employed labor force; i_{in} – domestic investment per unit of labor; i_f – foreign investment per unit of labor; k_{gov} – capital intensity of the public sector; g – taxes per unit of labor; d_{gov} – depreciation ratio for public capital; t_x – net government international transfers; m – total savings per unit of labor; s – accumulation rate; n_N – land factor per unit of labor; i_{out} – external investment per unit of labor.

Provided that i_{in}, i_{out}, i_f, tx, k_{rd} and g are exogenous, their stationary values will be as in *Tbl. 2*.

The next step is to find all the analytical solutions of the model components under the same condition on the exogenous parameters i_{in}, i_{out}, i_f, tx, k_{rd} і g (*Tbl. 3*).

Lower and upper bounds can be found using the formulas in *Tbl. 4*.

The approximate degree of risk will be calculated using the formulas in *Tbl. 5*.

Practical results of computer modeling of the numerical measure of economic risk. First of all, let's consider the risk of economic system evolution in the context of the above models in the time dimension (*Fig. 1*) based on data from [17].

The calculation of risk based on the results of economic growth modeling showed significant differences depending on the model chosen. The lowest level of risk for private capital was recorded in the multivariate author's model. This is explained by a much smaller number of critical and unrealistic assumptions compared to classical models, as the author's model integrates a wider range of factors without a rigid assumption of a constant savings rate or consumer preferences, which reduces the structural vulnerability of risk calculations.

In the Solow model, the risk was somewhat higher. This is due to its simplicity: the model assumes that population growth and technological progress are exogenous, which does not allow for internal shocks in economic development. The sharp increase in risk according to this model was particularly noticeable in the late 1980s and early 1990s, due to large-scale structural transformations of developing economies and the transition to market systems.

The Mankiw – Romer – Weil model demonstrated a level of risk comparable to the Solow model, due to its basis on the classic assumptions of the Solow model with the addition of human capital. This model showed an increase in risk in the early 2010s due to global structural changes in the labor market and the role of human capital in the context of digitalization.

The highest risk was obtained with the Ramsey – Cass – Koopmans model. The reason for this is its methodological approach, which defines savings as the difference between output and consumption. This concept is not realistic enough for practical application, as it ignores the institutional and market constraints that govern the dynamics of savings in modern economies. The growth of risk under this model became particularly pronounced in the late 1990s and throughout the 2000s, peaking in 2008 under the influence of the global financial crisis..

Almost similar results were obtained for the Ukrainian economy in terms of ranking models by risk (*Fig. 2*).

The calculation of the economic growth risk for Ukraine showed that according to the Solow, Ramsey – Cass – Koopmans, and Mankiw – Romer – Weil models, there is an increasing risk dynamics: in the Solow model due to its dependence on exogenous technological progress and population growth rates, which in the Ukrainian context demonstrate instability; in the Mankiw – Romer – Weil model due to high sensitivity to changes in the level of human capital accumulation, which in Ukraine is characterized by an outflow of skilled labor; in the Ramsey – Cass – Koopmans model, the risk increases the most, as this model is based on the assumption that savings are defined as the difference between output and consumption, which does not correspond to the realities of the Ukrainian economy with its low propensity to save and limited investment potential. Instead, the author's multidimensional model showed minimal downside risk due to the inclusion of a wide range of macroeconomic indicators (including actual capital flows, productivity dynamics, and institutional changes) without strict exogenous assumptions, which allowed for a more adequate reflection of the real structure and adaptability of the Ukrainian economy to internal and external shocks.

Table 2

k_{pr}^{*}	k_{gov}^{*}	m^{*}	h^{*}
$\left[\dfrac{i_{in}^{*}+i_{f}^{*}}{d_{pr}+n}\right]$	$\left[\dfrac{g+tx^{*}}{d_{gov}+n}\right]$	$\left[\dfrac{sAk_{pr}^{*\,\alpha}k_{gov}^{*\,\beta}h^{*\gamma}n_{N}^{\varphi}-(g^{*}+i_{in}^{*}+i_{out}^{*})}{n}\right]$	$\left[\dfrac{Blk_{rd}^{*\,\nu}}{n}\right]$

Table 3

Index	Formula
$k_{pr}(t)$	$\left[\left(k_{pr0}-\dfrac{i_{in}^{*}+i_{f}^{*}}{d_{pr}+n}\right)e^{-(d_{pr}+n)t}+\dfrac{i_{in}^{*}+i_{f}^{*}}{d_{pr}+n}\right]$
$k_{gov}(t)$	$\left[\left(k_{gov0}-\dfrac{g^{*}+tx^{*}}{d_{pr}+n}\right)e^{-(d_{gov}+n)t}+\dfrac{g^{*}+tx^{*}}{d_{gov}+n}\right]$
$m(t)$	$e^{-nt}\left[m_{0}+\int_{0}^{t}\left(sAk_{pr}^{\alpha}\,k_{gov}^{\beta}\,h^{\gamma}\,n_{N}^{\varphi}-\left(g+nm+i_{in}+i_{out}\right)\right)e^{-nr}dr\right]$
$h(t)$	$\left[\left(h_{0}-\dfrac{Blk_{rd}^{*\,\upsilon}}{n}\right)e^{-nt}+\dfrac{Blk_{rd}^{*\,\upsilon}}{n}\right]$

Table 4

Index	Lower limit	Upper limit
$k_{pr}(t)$	$\left[\left(k_{pr0}-\dfrac{i_{in}^{*}+i_{f}^{*}}{d_{pr}+n}\right)e^{-(d_{pr}+n)t}\right]$	$\left[k_{pr0}\,e^{-(d_{pr}+n)t}+\dfrac{i_{in}^{*}+i_{f}^{*}}{d_{pr}+n}\right]$
$k_{gov}(t)$	$\left[\left(k_{gov0}-\dfrac{g^{*}+tx^{*}}{d_{gov}+n}\right)e^{-(d_{gov}+n)t}\right]$	$\left[k_{gov0}\,e^{-(d_{gov}+n)t}+\dfrac{g^{*}+tx^{*}}{d_{gov}+n}\right]$
$m(t)$	$e^{-nt}\left[m_{0}+\int_{0}^{t}\left(\begin{matrix}sAk_{pr}^{\alpha}\,k_{gov}^{\beta}\,h^{\gamma}n_{N}^{\varphi}-\\-(g+nm+i_{in}+i_{out})\end{matrix}\right)e^{-nr}dr\right]-m^{*}$	$e^{-nt}\left[m_{0}+\int_{0}^{t}\left(\begin{matrix}sAk_{pr}^{\alpha}\,k_{gov}^{\beta}\,h^{\gamma}n_{N}^{\varphi}-\\-(g+nm+i_{in}+i_{out})\end{matrix}\right)e^{-nr}dr\right]+$ $-e^{-nt}m^{*}$
$h(t)$	$\left[\left(h_{0}-\dfrac{Blk_{rd}^{*\,\upsilon}}{n}\right)e^{-nt}\right]$	$\left[h_{0}\,e^{-nt}+\dfrac{Blk_{rd}^{*\,\upsilon}}{n}\right]$

The instability of the economic situation in Ukraine and difficult-to-predict processes, such as political crises, military actions, sharp changes in the external economic environment and exchange rate fluctuations, have significantly affected the results of risk calculation using classical models. In particular, the Solow model, due to its simplified structure, does not take into account the effects of institutional and political instability, which leads to an underestimation of real risks. Despite the integration of human capital, the Mankiw – Romer – Weil model does not reflect the high volatility and migration processes characteristic of Ukraine. The Ramsey – Cass – Koopmans model was the most sensitive, as its assumptions about

Table 5

Index	Formula
$R(k_{pr})$	$\sqrt{\dfrac{\left[k_{pr0}\, e^{-(d_{pr}+n)t} + \dfrac{i_{in}^* + i_f^*}{d_{pr}+n}\right]}{\left[\left(k_{pr0} - \dfrac{i_{in}^* + i_f^*}{d_{pr}+n}\right) e^{-(d_{pr}+n)t}\right]}}$
$R(k_{gov})$	$\sqrt{\dfrac{\left[k_{gov0}\, e^{-(d_{gov}+n)t} + \dfrac{g^* + tx^*}{d_{gov}+n}\right]}{\left[\left(k_{gov0} - \dfrac{g^* + tx^*}{d_{gov}+n}\right) e^{-(d_{gov}+n)t}\right]}}$
$R(m)$	$\sqrt{\dfrac{e^{-nt}\left[m_0 + \int_0^t \left(sAk_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - (g + nm + i_{in} + i_{out})\right) e^{-nr} dr\right] + e^{-nt} m^*}{e^{-nt}\left[m_0 + \int_0^t \left(sAk_{pr}^{\alpha} k_{gov}^{\beta} h^{\gamma} n_N^{\varphi} - (g + nm + i_{in} + i_{out})\right) e^{-nr} dr\right] - m^*}}$
$R(h)$	$\sqrt{\dfrac{\left[h_0\, e^{-nt} + \dfrac{Blk_{rd}^{*\upsilon}}{n}\right]}{\left[\left(h_0 - \dfrac{Blk_{rd}^{*\upsilon}}{n}\right) e^{-nt}\right]}}$

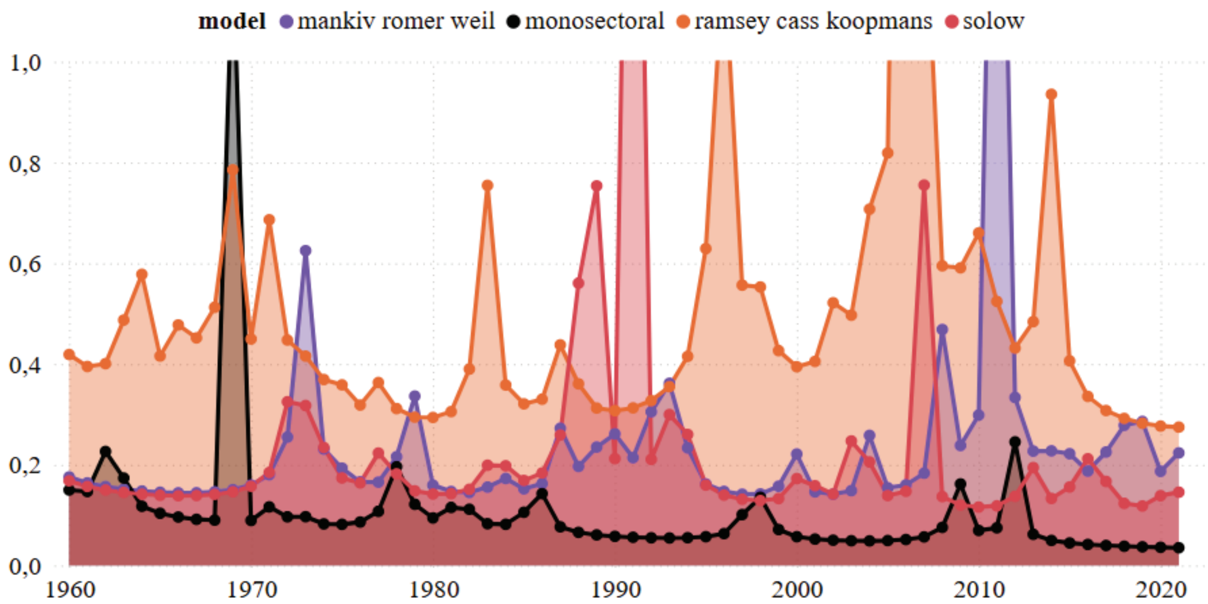

Fig. 1. Average risk by economic growth model in the time dimension based on the authors' calculations

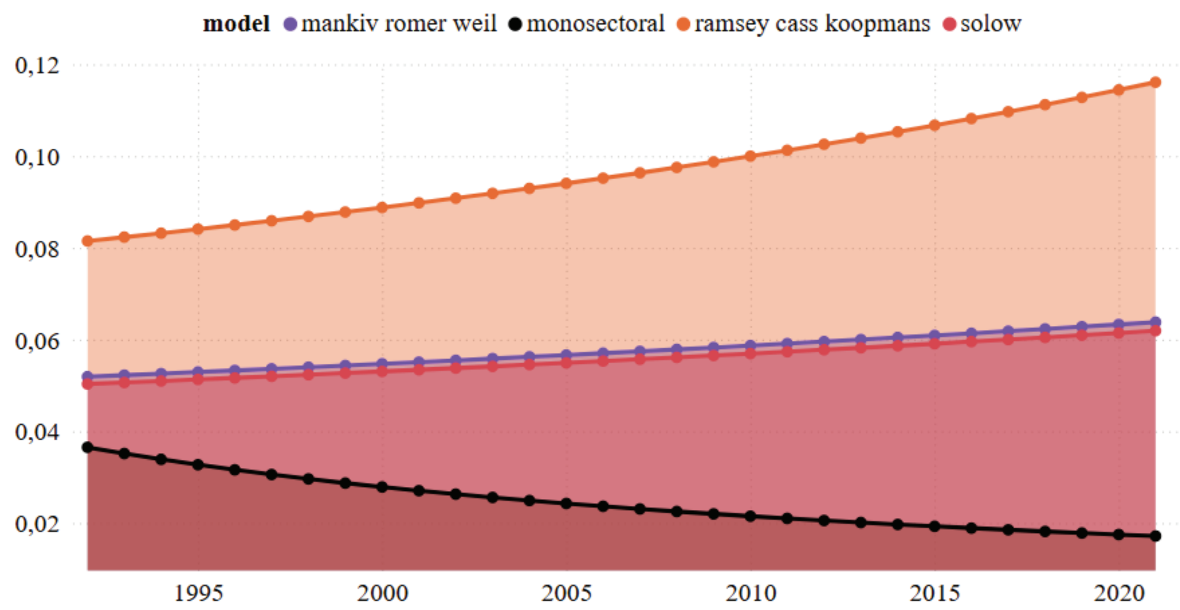

Fig. 2. Risk in terms of economic growth models in the time dimension for the Ukrainian economy based on the authors' calculations

households optimizing consumption and savings in a stable environment contradict the actual behavior of economic agents in a crisis, which significantly increased the assessed risk. Thus, it is the structural instability and multifactorial nature of Ukraine's economic dynamics that have become the key reasons for the overestimation of risk by these models.

Let's take a closer look at the average risk by income group (*Fig. 3*).

The highest level of risk in all income groups (maximum 0.98 in low-income countries) is due to a methodological feature of the model: the definition of savings as the difference between output and consumption assumes stable household behavior, stable intertemporal optimization, and no shocks. In low-income countries, these assumptions are the most disconnected from reality due to high poverty rates, income instability, and the lack of effective mechanisms for intertemporal consumption reassessment. In high-income countries, the risk is lower (0.31) because capital markets are more developed, but the model is still too sensitive to fluctuations in consumption, especially during crises, which explains the results obtained.

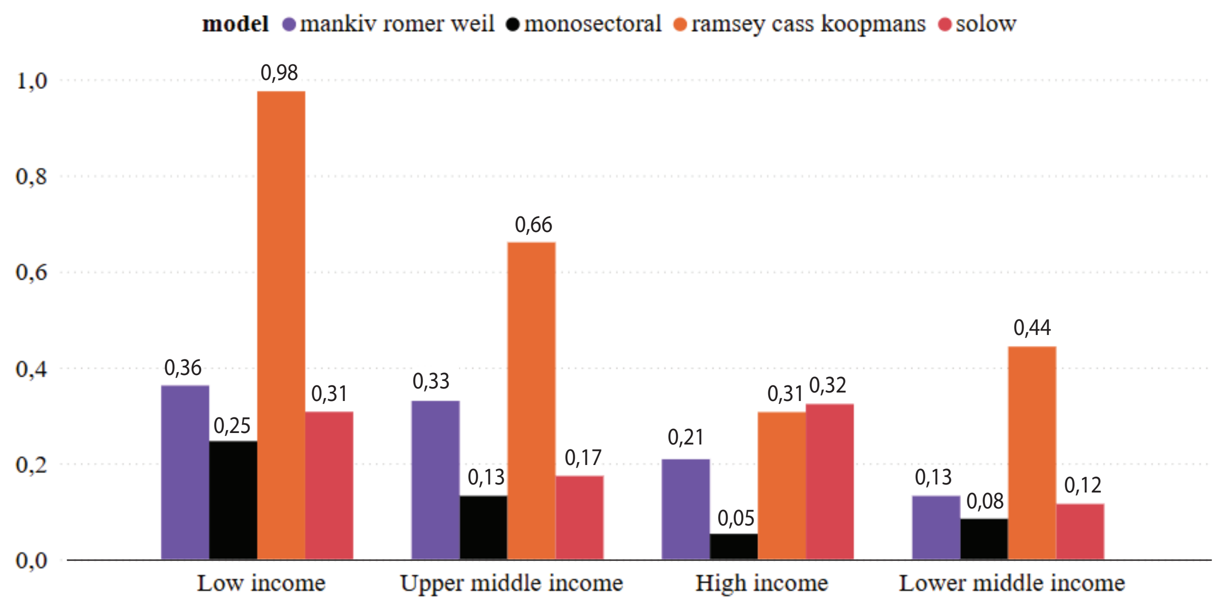

Fig. 3. Average risk by economic growth model by income group based on the authors' calculations

The Mankiw – Romer – Weil model demonstrates moderate levels of risk (0.36 in low-income countries and 0.13–0.33 in middle- and high-income countries) due to the fact that the model takes into account human capital, which partially stabilizes growth forecasts. However, in low-income countries, there is an increased risk due to poor educational development, outflow of skilled workers, and low quality of human capital, which makes the model parameters unstable. In higher-income countries, the risk is reduced due to better educational and innovation infrastructure.

The Solow model shows similar or slightly lower levels of risk compared to the Mankiw – Romer – Weil model: 0.31 in low-income countries, 0.17 in upper middle-income countries, because in developed economies, growth factors go far beyond physical capital alone, and simplifying the model leads to a loss of accuracy. Thus, the absence of human capital reduces the risk only conditionally, as the model loses its sensitivity to real structural changes in the economy.

The lowest risk levels (e. g., 0.05 for highly developed countries) are due to the fact that the model takes into account actual data and the multifactorial structure of the economy without making rigid exogenous assumptions about the sustainability of technological progress or the savings rate. It integrates macroeconomic indicators, institutional factors, productivity dynamics, and market realities, which allows it to accurately reflect the economic situation even in countries with unstable incomes. This is what makes the risk of this model minimal and slightly declining over time, as it is structurally more flexible and realistic.

Overall, we have similar results when considering risk by geography (*Fig. 4*).

North Africa and the Middle East have the highest average economic growth risk scores, which is directly related to their structure of resource-based economies, focused mainly on oil and gas exports. This concentration of production and revenues in the commodity sector makes the economic dynamics of these regions highly dependent on global energy prices, which are highly volatile, especially during global crises or geopolitical instability. In addition, a significant share of government spending is financed by revenues from resource exports, which creates additional fiscal risks in the event of lower prices, reduced production, or international sanctions, which together form the increased average risk indicators for these countries in the modeling results.

Fig. 5 – Fig. 7 show the modeling results for the Ukrainian economy. In general, they are characterized by similar trends for all models: there is a downward trend in capital intensity and an upward trend in human capital per unit of labor.

The decline in capital intensity is explained by a decrease in investment in physical capital due to ongoing economic instability, devaluation of the national currency, and limited access to external financing, which hinders the renewal of fixed assets.

The growth of human capital per unit of labor is driven, on the one hand, by a decline in the number of employed due to demographic and migration factors, and, on the other hand, by a gradual increase in the average level of education and professional qualifications of those who remain in the labor market, which structurally raises the human capital indicator. Such

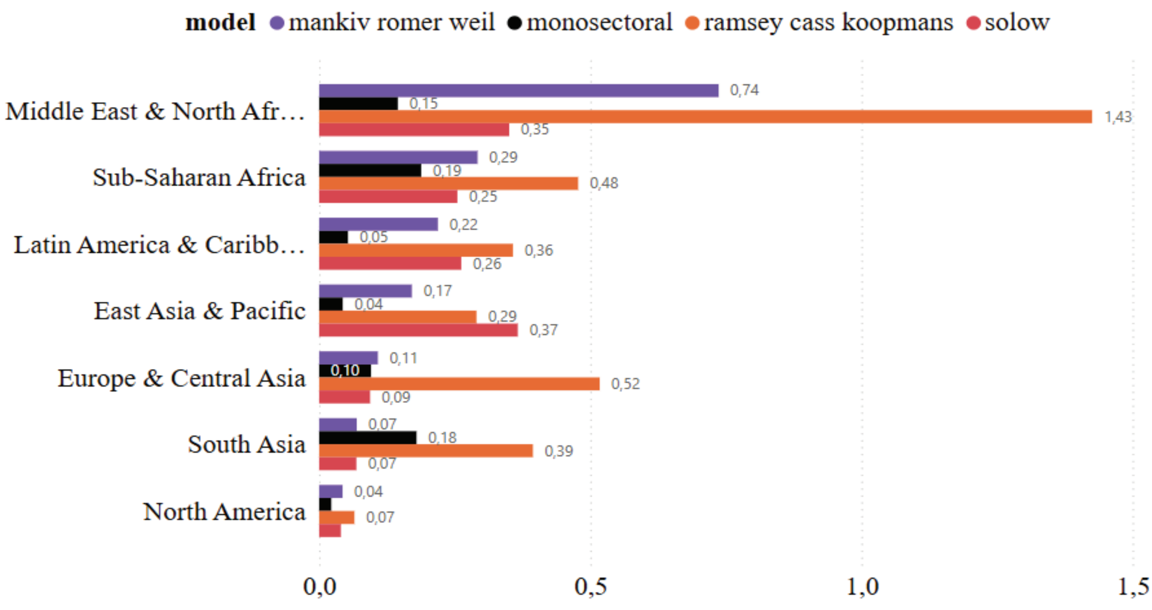

Fig. 4. Average risk by economic growth model by geographic location based on the authors' calculations

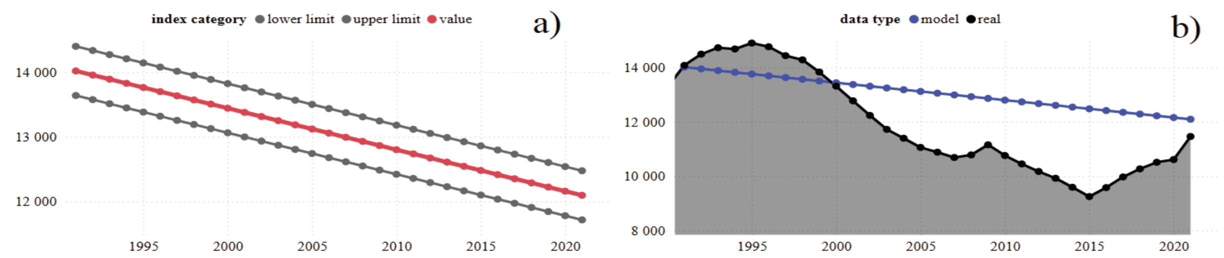

Fig. 5. Model values of capital intensity and its limits (a), comparison of model and real values of capital intensity (b) for the Solow model based on the authors' calculations

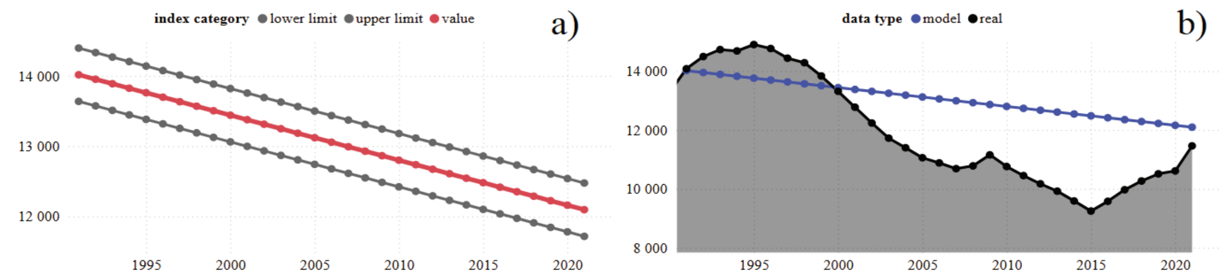

Fig. 6. Model values of capital intensity and its limits (a), comparison of model and real values of capital intensity (b) for the Ramsey – Cass – Koopmans model based on the authors' calculations

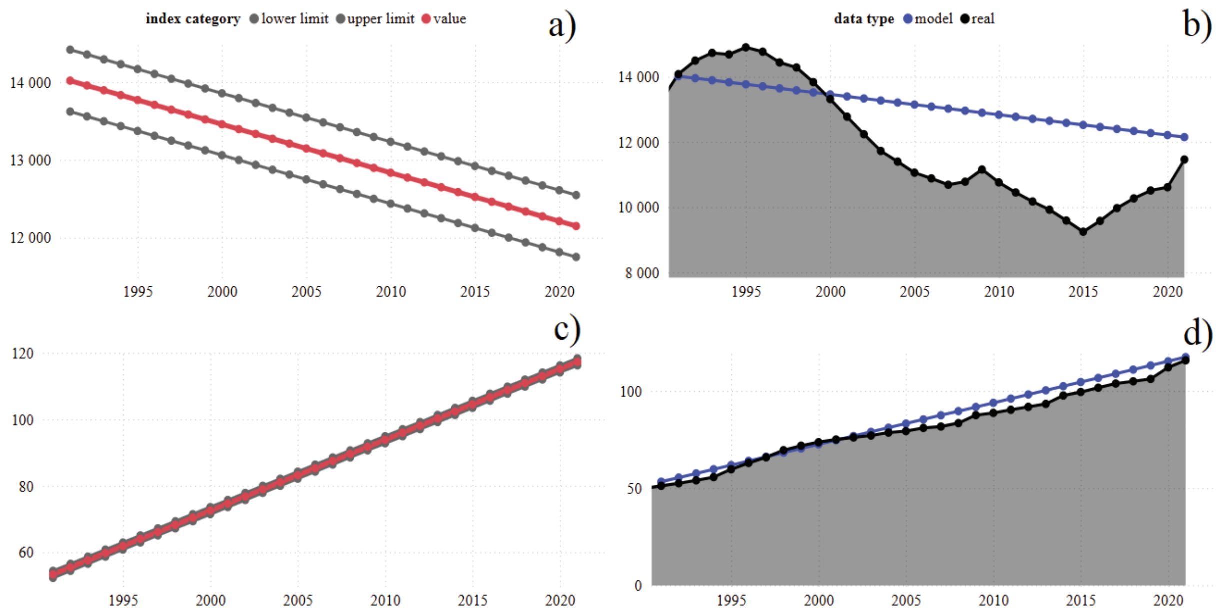

Fig. 7. Model values of the main indicators and their limits (a,c), comparison of model and real values of the main indicators (b, d) for the Mankiw – Romer – Weil model based on the authors' calculations

results are logical in a transformational economy, where the production base is slowly degrading amid investment stagnation, but at the same time, the positive dynamics of human potential development continues to be observed due to educational reforms, adaptation to market conditions, and more intensive use of knowledge in the value added structure.

As for the author's multivariate model with more parameters (*Fig. 8*), in addition to the above trends of downward dynamics of private capital intensity and growth of human capital, we can also observe an increase in public capital intensity. This is due to the fact that the model takes into account separately the state capital, investments in infrastructure, defense, security and other public assets, the share of which in the Ukrainian economy is constantly growing in response to the challenges of the war and the implementation of international assistance programs.

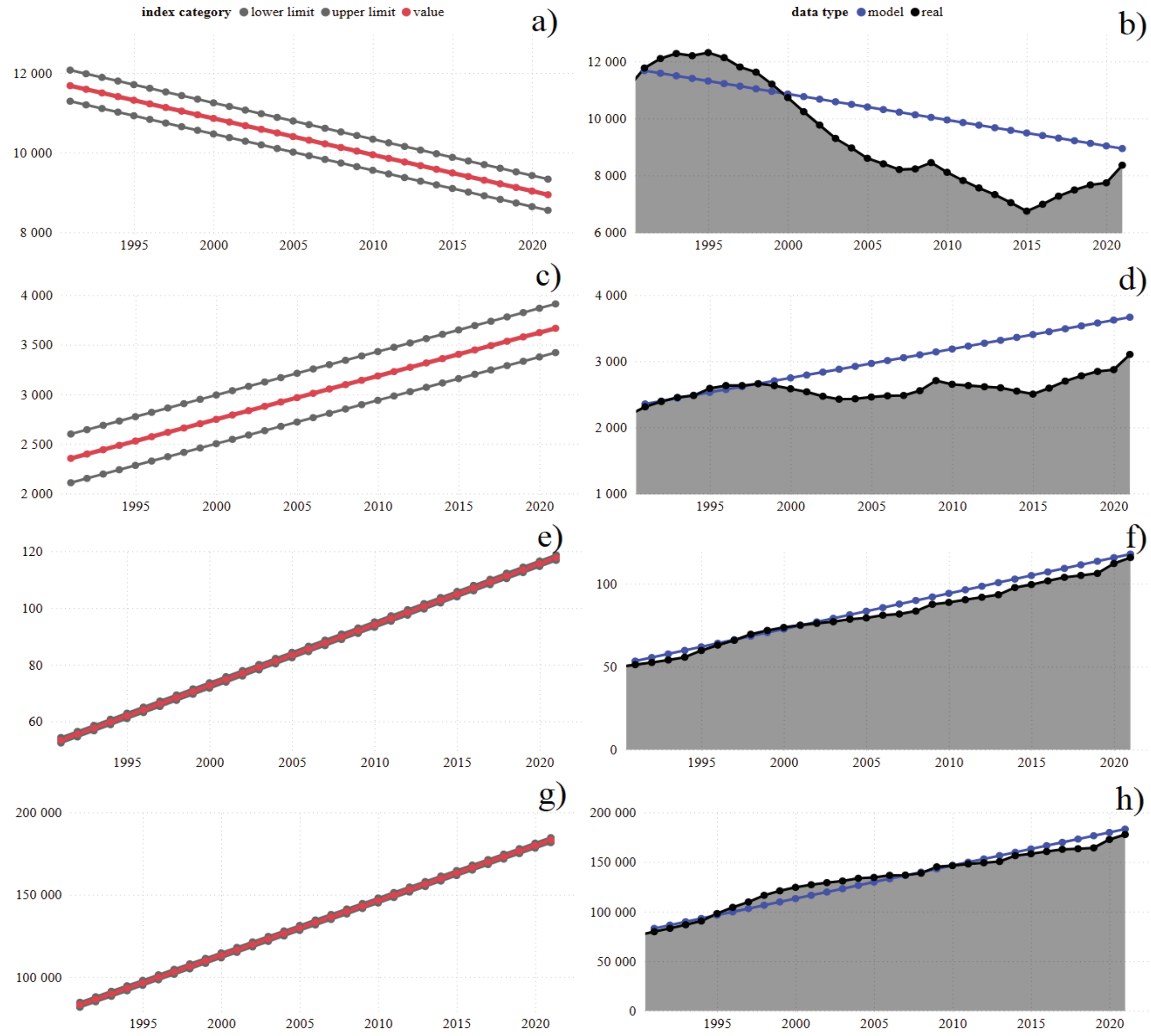

Fig. 8. Model values of the main indicators and their limits (a, c, e, g), comparison of model and real values of the main indicators (b, d, f, h) for the author's model based on the authors' calculations

Instead, private capital intensity continues to fall similarly to other models due to the limited ability of domestic businesses to invest in physical capital in an environment of high risks, inflation, and lack of financing.

At the same time, total savings per unit of labor show an upward trend, reflecting both a reduction in the number of employed (a decrease in the denominator) and an increased role of public and institutional savings, in particular in the form of external financial support and fiscal consolidation, which partially compensates for the lack of private savings in the economy.

Next, we compare the risk of economic growth for Ukraine with certain groups of countries (*Fig. 9* – *Fig. 12*). To begin with, let's compare with the countries of Eastern Europe (*Fig. 9*).

According to Fig. 9, Ukraine faced a much lower level of economic growth risk in the 1990s compared to most of its neighboring countries, such as Poland, Romania, Hungary, or Slovakia. This is explained by the fact that in the early 1990s the Ukrainian economy remained largely inertial, with Soviet production linkages, centralized planning, low openness and low foreign trade integration, which temporarily reduced the estimated risk in models sensitive to market shocks and structural changes.

At the same time, Central European countries were already actively pursuing economic reforms, price liberalization, privatization, and market opening, which was accompanied by high uncertainty and increased risk in the short term. However, in the following years, the situation changed dramatically: in

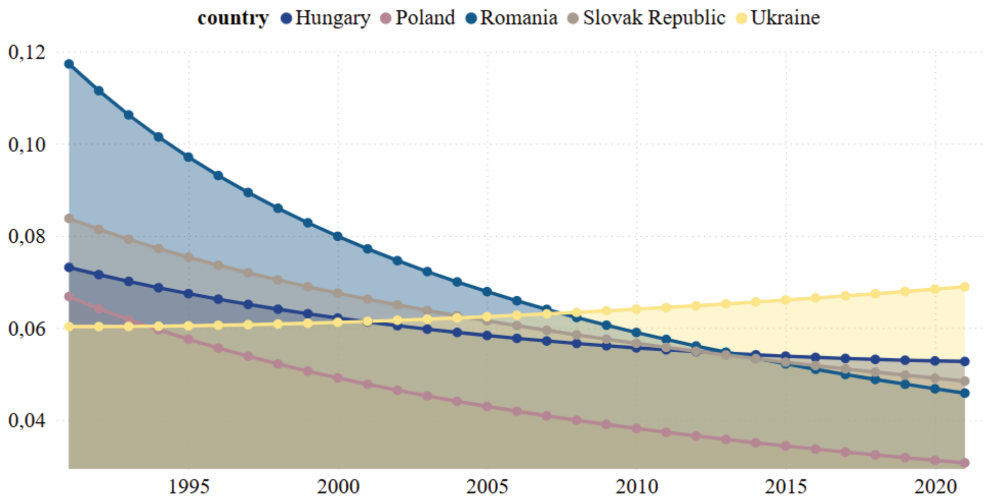

Fig. 9. Average risk in the sample of Eastern European countries in the time dimension based on the authors' calculations

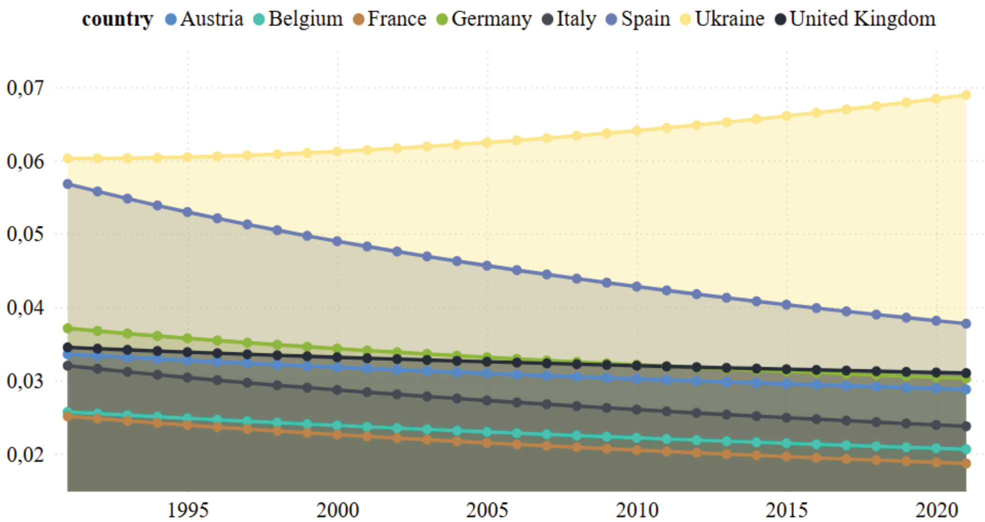

Fig. 10. Average risk in the sample of Western European countries in the time dimension based on the authors' calculations

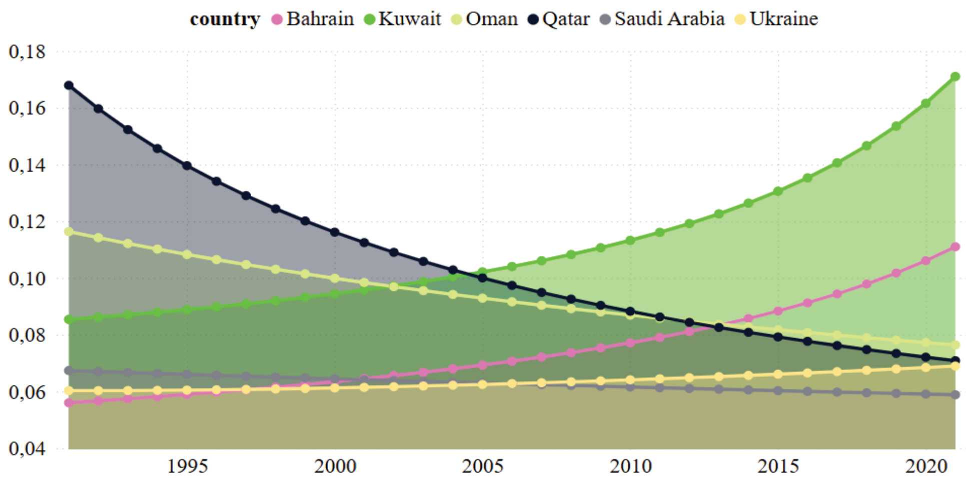

Fig. 11. Average risk in the sample of oil-producing countries in the time dimension based on the authors' calculations

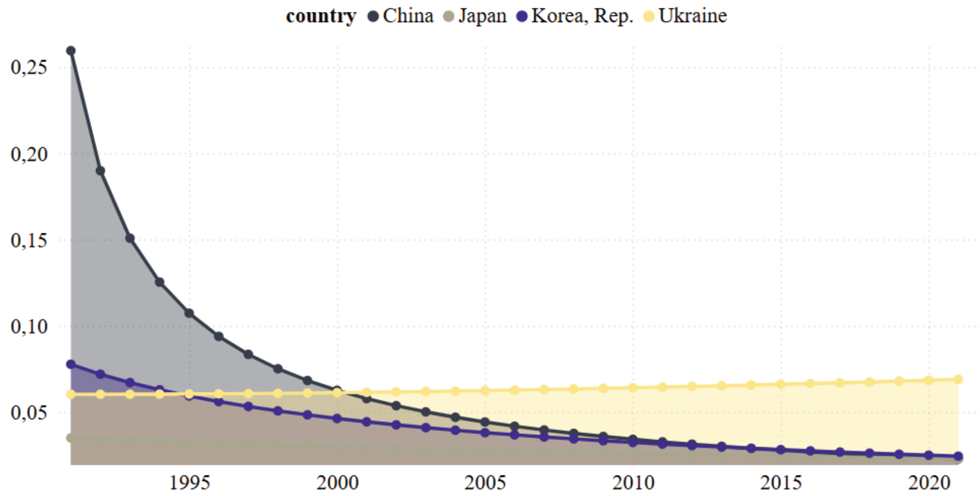

Fig. 12. Average risk in the sample of developed Asian countries in the time dimension based on the authors' calculations

Ukraine, risk increased due to the lack of systemic market transformation, chronic political instability, weak property institutions, and numerous crises, while in Poland, Hungary, Slovakia, and Romania, reforms contributed to increased economic efficiency, stabilized macroeconomic indicators, and significantly reduced economic risks, and the countries became EU members, which also had a significant impact.

Compared to Ukraine, Western European countries have a much lower level of economic growth risk due to a combination of structural, economic and institutional factors. First, these countries are characterized by a more stable political system, which creates a predictable environment for investment and long-term planning. Secondly, Western European countries have developed infrastructure, high quality human capital and innovative potential, which ensures the resilience of their economies even in times of global crises. Third, an effective system of financial sector regulation and social guarantees minimizes volatility and increases the economy's adaptability to external and internal shocks. In addition, the level of economic diversification in these countries is much higher, which reduces dependence on individual sectors and commodities, unlike Ukraine, where the economy often remains more highly specialized and vulnerable to external fluctuations.

However, even among developed countries, there are significant differences in the level of economic growth risk. For example, Spain has a much higher level of risk compared to other Western European countries, although this risk is gradually decreasing. This is due to a number of factors, including historically high unemployment, especially among young people, which limits domestic demand and the stability of the social system, the Spanish economy remains more dependent on certain sectors, such as tourism, which is vulnerable to external shocks such as pandemics or fluctuations in global demand, and problems in the banking sector and high public debt in certain periods have increased uncertainty and risks to the economy. However, thanks to structural reforms, improved institutional efficiency, and increased competitiveness, these risks are gradually decreasing..

A comparison with oil-producing countries (*Fig. 11*) shows that they, on average, have a much higher level of economic growth risk than Ukraine. This is due to the high dependence of their economies on world oil prices, which are highly volatile and subject to geopolitical factors.

In particular, in countries such as Kuwait and Bahrain, the risk increases over time due to declining oil reserves, as well as the need to diversify the economy in the context of a gradual transition to sustainable development and a decrease in global demand for fossil fuels.

In addition, oil countries often face challenges in managing large government revenues, which can lead to instability in the financial system and political risks. All of this makes it more difficult to maintain stable economic growth and increases the overall level of risk compared to Ukraine's more diversified economy. Moreover, even Ukraine's political instability does not offset all risks.

Among the oil-producing countries analyzed, only Saudi Arabia demonstrates a lower level of economic growth risks compared to Ukraine. This is due to the scale of its economy, high oil reserves, and a more effective resource management strategy that ensures more stable state budget revenues.

Risks to Japan's economy (*Fig. 12*) are roughly at the level of Western European countries, indicating a high degree of stability and predictability in its economic development. This is due to its strong institutional system, technological development, diversified economy, and effective financial management. By contrast, the risks to China's economy (Fig. 12) in the 1990s were more than five times higher than those of Ukraine, reflecting significant instability, a transitional economy, and rapid but uneven development. However, over the past decades, China has significantly strengthened its economic position: it has strengthened its institutional mechanisms, developed its industry and financial sector, and expanded its domestic market. As a result, the level of risks to China's economy has almost reached the level of risks to Japan's economy, demonstrating a marked decrease in instability and an increase in economic resilience.

CONCLUSIONS

The article explores the theoretical foundations and practical methods for determining the steady state of the economy, analyzing its growth and assessing risks. Classical models of economic development are considered and a quantitative risk assessment based on deviations from the equilibrium trajectory is proposed, which helps to assess the sustainability of the economy and the impact of risks on investment and stability. As a methodology for assessing the degree of risk, the author proposes an approach of coordinate calculation of the ratio of the maximum and minimum limits of the analytical solution of economic growth models.

The practical calculation of risk based on different models of economic growth revealed significant differences. The lowest level of risk for private capital was recorded in the author's multivariate model, which is explained by fewer critical assumptions and the consideration of a wider range of factors without rigid assumptions about the savings rate or consumer preferences, which reduces the vulnerability of the estimates. In the Solow model, the risk was higher due to its simplicity and exogenous population and technology growth rates, which does not take into account internal shocks. This model saw a particularly sharp increase in risk in the late 1980s and early 1990s due to structural changes in economies transitioning to the market. The Mankiw – Romer – Weil model showed a risk close to Solow's, as it is based on classical assumptions with the addition of human capital. The risk increased in the early 2010s due to labor market transformations and digitalization. The Ramsey – Cass – Koopmans model yielded the highest risk, due to the methodology that defines savings as the difference between output and consumption, ignoring institutional and market constraints. The risk in this model grew in the 1990s and 2000s and peaked in 2008 under the influence of the global financial crisis..

The instability of the economic situation in Ukraine and difficult-to-predict processes, such as political crises, military actions, sharp changes in the external economic environment and exchange rate fluctuations, negatively affected the results of the economic risk calculation. Compared to neighboring countries that were undergoing complex reforms with high uncertainty at the beginning of the modeling period, they have significantly reduced their risks due to reforms and integration into the EU. Western European countries and other highly developed countries have a much lower risk due to stable policies, developed infrastructure, high quality human capital, and diversified economies. Oil-producing countries have a higher risk due to their dependence on volatile oil prices, declining reserves, and the need for diversification. Many countries in Africa, Asia, and Oceania are at high risk due to poverty levels, political risks, poorly differentiated economies, etc.

In future studies, the NBU plans to expand the use of the coordinate approach to assessing economic growth risk by incorporating more complex models that take into account stochastic factors and structural changes in the economy. Particular attention will be paid to analyzing the impact of external shocks and uneven sectoral development on the formation of growth trajectories and their stability. In addition, it is planned to integrate empirical data to validate analytical boundaries and refine the parameters of risk assessments in order to increase their practical significance for forecasting economic dynamics. ■

BIBLIOGRAPHY

1. Вітлінський В. В., Великоіваненко Г. І. Ризикологія в економіці та підприємництві : монографія. Київ : КНЕУ, 2004. 480 с.
2. Kohlscheen E., Nakajima J. Steady-State Growth. *International Finance*. 2021. Vol. 24. Iss. 1. P. 40–52. DOI: https://doi.org/10.1111/infi.12386
3. Mautinović I., Ulanowicz R. E., Vlah D. Exploring theoretical conditions for a steady-state global economy: A simulation model. *The Anthropocene Review*. 2023. Vol. 11. Iss. 1. DOI: https://doi.org/10.1177/20530196231170369
4. Andrle M., Hunt B. L. Model-Based Globally-Consistent Risk Assessment. *International Monetary Fund*, 2020. 28 p. DOI: https://doi.org/10.5089/9781513536460.001
5. Wang W., Wei K., Kubatko O. et al. Economic Growth and Sustainable Transition: Investigating Classical and Novel Factors in Developed Countries. *Sustainability*. 2023. Vol. 15. No. 16. Art. 12346. DOI: https://doi.org/10.3390/su151612346

6. Klimek P., Poledna S., Thurner S. Quantifying economic resilience from input-output susceptibility to improve predictions of economic growth and recovery. *Nature Communications*. 2019. Vol. 10. No. 1. DOI: https://doi.org/10.1038/s41467-019-09357-w
7. Solow R. M. A Contribution to the Theory of Economic Growth. *The Quarterly Journal of Economics*. 1956. Vol. 70. Iss. 1. P. 65–94. DOI: https://doi.org/10.2307/1884513
8. Solow R. M. Technical Change and the Aggregate Production Function. *The Review of Economics and Statistics*. 1957. Vol. 39. No. 3. P. 312–320. DOI: https://doi.org/10.2307/1926047
9. Swan T. W. Economic Growth and Capital Accumulation. *Economic Record*. 1956. Vol. 32. Iss. 2. P. 334–361. DOI: https://doi.org/10.1111/j.1475-4932.1956.tb00434.x
10. Ramsey F. P. A Mathematical Theory of Saving. *The Economic Journal*. 1928. Vol. 38. No. 152. P. 543–559. DOI: https://doi.org/10.2307/2224098
11. Cass D. Optimum Growth in an Aggregative Model of Capital Accumulation. *The Review of Economic Studies*. 1965. Vol. 32. Iss. 3. P. 233–240. DOI: https://doi.org/10.2307/2295827
12. Koopmans T. C. On the Concept of Optimal Economic Growth. Cowles Foundation for Research in Economics, Yale University. Discussion Paper. 1963. No. 163. URL: https://elischolar.library.yale.edu/cowles-discussion-paper-series/392
13. Mankiw N. G., Romer D., Weil D. N. A Contribution to the Empirics of Economic Growth. *The Quarterly Journal of Economics*. 1992. Vol. 107. Iss. 2. P. 407–437. DOI: https://doi.org/10.2307/2118477
14. Kolyada Yu., Poznyak S., Ramskyi A., Shevchenko S. Adaptation Applying of Economic Growth Theoretical Models. *Digital Economy Concepts and Technologies*. Workshop 2024. 2024. P. 22–35. URL: https://ceur-ws.org/Vol-3665/paper3.pdf
15. Коляда Ю., Позняк С. Адаптивна модель економічного зростання для відкритої економіки. *Національні інтереси України*. 2024. № 2. DOI: https://doi.org/10.52058/3041-1793-2024-2(2)-267-286
16. Позняк С., Коляда Ю. Адаптивна модель зростання відкритої економіки. *Теорія модернізації в контексті сучасної світової науки* : матеріали III Міжнародної наукової конференції МЦНД (м. Кривий Ріг, 02.08.2024). Кривий Ріг, 2024. С. 87–92. DOI: https://doi.org/10.62731/mcnd-02.08.2024.001
17. World Bank. URL: https://data.worldbank.org/

REFERENCES

Andrle, M., and Hunt, B. L. Model-Based Globally-Consistent Risk Assessment. *International Monetary Fund*, 2020. DOI: https://doi.org/10.5089/9781513536460.001

Cass, D. "Optimum Growth in an Aggregative Model of Capital Accumulation". *The Review of Economic Studies*, vol. 32, no. 3 (1965): 233-240. DOI: https://doi.org/10.2307/2295827

Klimek, P., Poledna, S., and Thurner, S. "Quantifying economic resilience from input-output susceptibility to improve predictions of economic growth and recovery". *Nature Communications*, vol. 10, no. 1 (2019). DOI: https://doi.org/10.1038/s41467-019-09357-w

Kohlscheen, E., and Nakajima, J. "Steady-State Growth". *International Finance*, vol. 24, no. 1 (2021): 40-52. DOI: https://doi.org/10.1111/infi.12386

Koliada, Yu., and Pozniak, S. "Adaptyvna model ekonomichnoho zrostannia dlia vidkrytoi ekonomiky" [Adaptive Model of Economic Growth for an Open Economy]. *Natsionalni interesy Ukrainy*, no. 2 (2024). DOI: https://doi.org/10.52058/3041-1793-2024-2(2)-267-286

Kolyada, Yu. et al. "Adaptation Applying of Economic Growth Theoretical Models". *Digital Economy Concepts and Technologies*. Workshop 2024. 2024. https://ceur-ws.org/Vol-3665/paper3.pdf

Koopmans, T. C. "On the Concept of Optimal Economic Growth. Cowles Foundation for Research in Economics, Yale University". *Discussion Paper*. 1963. https://elischolar.library.yale.edu/cowles-discussion-paper-series/392

Mankiw, N. G., Romer, D., and Weil, D. N. "A Contribution to the Empirics of Economic Growth". *The Quarterly Journal of Economics,* vol. 107, no. 2 (1992): 407-437. DOI: https://doi.org/10.2307/2118477

Mautinovic, I., Ulanowicz, R. E., and Vlah, D. "Exploring theoretical conditions for a steady-state global economy: A simulation model". *The Anthropocene Review,* vol. 11, no. 1 (2023). DOI: https://doi.org/10.1177/20530196231170369

Pozniak, S., and Koliada, Yu. "Adaptyvna model zrostannia vidkrytoi ekonomiky" [Adaptive Growth Model of an Open Economy]. *Teoriia modernizatsii v kontekst suchasnoi svitovoi nauky*. Kryvyi Rih, 2024. 87-92. DOI: https://doi.org/10.62731/mcnd-02.08.2024.001

Ramsey, F. P. "A Mathematical Theory of Saving". *The Economic Journal*, vol. 38, no. 152 (1928): 543-559. DOI: https://doi.org/10.2307/2224098

Solow, R. M. "A Contribution to the Theory of Economic Growth". *The Quarterly Journal of Economics*, vol. 70, no. 1 (1956): 65-94. DOI: https://doi.org/10.2307/1884513

Solow, R. M. "Technical Change and the Aggregate Production Function". *The Review of Economics and Statistics*, vol. 39, no. 3 (1957): 312-320. DOI: https://doi.org/10.2307/1926047

Swan, T. W. "Economic Growth and Capital Accumulation". *Economic Record*, vol. 32, no. 2 (1956): 334-361. DOI: https://doi.org/10.1111/j.1475-4932.1956.tb00434.x

Vitlinskyi, V. V., and Velykoivanenko, H. I. *Ryzykolohiia v ekonomitsi ta pidpryiemnytstvi* [Riskology in Economics and Entrepreneurship]. Kyiv: KNEU, 2004.

Wang, W. et al. "Economic Growth and Sustainable Transition: Investigating Classical and Novel Factors in Developed Countries". *Sustainability*, art. 12346, vol. 15, no. 16 (2023). DOI: https://doi.org/10.3390/su151612346

World Bank. https://data.worldbank.org/