MODELING AND FORECASTING UKRAINE'S POPULATION BY TIME SERIES USING THE MATLAB ECONOMETRICS TOOLBOX

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Kovalova K. O., Misiura I. Y. Modeling and Forecasting Ukraine's Population by Time Series Using the Matlab Econometrics Toolbox

The article deals with modeling and forecasting the population of Ukraine by time series. It is shown that time series analysis is a complex, multicomponent econometric task which does not have a universal approach to its solution. This is due both to the diversity of methods of and approaches to time series analysis which were developed over time and to the specifics of time series data. For example, the authors of the article worked with a univariate nonstationary time series, therefore, the approaches and methods presented in the article are not recommended for time series with different properties. The article has an enormous practical value, since it discusses in detail issues of computer modeling of tasks of the kind. The carried out analysis of the literature has shown the relevance of the problems considered, among which particular attention should be paid to the choice of the ARIMA model, data visualization, and forecast accuracy.

Keywords: time series, nonstationarity, ARIMA models, Econometrics Toolbox, MATLAB.

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Ковальова К. О., Місюра Є. Ю. Моделювання і прогнозування населення України за допомогою часових рядів з використанням економетричного інструментального комплексу MATLAB

Статтю присвячено моделюванню та прогнозуванню населення України за допомогою часових рядів. Показано, що аналіз часових рядів є складною, багатокомпонентною економетричною задачею, що не має універсального підходу до її розв'язання. Це пов'язано як з різноманітністю методів і алгоритмів до аналізу часових рядів, розроблених за весь час, так і зі специфікою цих часових рядів. Так, наприклад, автори статті працювали з одновимірним нестаціонарним тимчасовим рядом, тому підходи і методи, наведені в цій статті, не рекомендується застосовувати до часових рядах, що володіють іншими властивостями. Стаття має колосальну практичну цінністю, оскільки в ній детально розглянуто питання комп'ютерного моделювання такого роду задач. Аналіз літератури показав актуальність цього питання на сьогодні. При цьому на перший план виходять такі важливі питання, як вибір моделі ARIMA, візуалізація даних і точність прогнозу.

Ключові слова: тимчасові ряди, нестаціонарність, моделі ARIMA, Econometrics Toolbox, MATLAB.

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Ковалева Е. А., Мисюра Е. Ю. Моделирование и прогнозирование населения Украины с помощью временных рядов с использованием эконометрического инструментального комплекса MATLAB

Статья посвящена моделированию и прогнозированию населения Украины по средствам временных рядов. Показано, что анализ временных рядов является сложной, многокомпонентной эконометрической задачей, не имеющей универсального подхода к ее решению. Это связано как с разнообразием методов и подходов к анализу временных рядов, разработанных за все время, так и со спецификой данных временных рядов. Так, например, авторы статьи работали с одномерным нестационарным временным рядом, поэтому подходы и методы, приведенные в данной статье, не рекомендуется применять к временным рядам, обладающими другими свойствами. Статья обладает колоссальной практической ценностью, так как в ней подробно рассмотрены вопросы компьютерного моделирования такого рода задач. Анализ литературы показал актуальность этого вопроса на сегодняшний день. При этом на первый план выходят такие важные вопросы, как выбор модели ARIMA, визуализация данных и точность прогноза.

Ключевые слова: временные ряды, нестационарность, модели ARIMA, Econometrics Toolbox, MATLAB.

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Time series analysis is a complex, multi-step process. In general it comprises four main stages described below.

1. Choice of time series analysis method (frequency-domain and time-domain methods [1]; parametric and non-parametric methods [2]; linear and nonlinear methods; univariate and multivariate ones).

2. Research analysis of time series data (autocorrelation analysis to examine serial dependence [3]; spectral analysis to examine cyclic behavior which need not be related to seasonality [4]; separation into components representing trend, seasonality, slow and fast variation, and cyclical irregularity).

3. Curve fitting, which is the process of constructing a curve or mathematical function that has the best fit to a series of data points. At this stage a researcher may choose a model for time series data.

Time series models appear in many forms and represent various random processes. When modeling variations in the level of a process, their three main classes can be singled out: autoregressive (AR) models, integrated (I) models and moving average (MA) models. These three classes are linearly dependent on previous data points. Combinations of these models provide autoregressive moving average (ARMA) and autoregressive integrated moving average (ARIMA) models. Autoregressive fractional integrated moving average (ARFIMA) model generalizes the three above mentioned classes. Nonlinear dependence of the level of a time series on previous data points is of interest, to some extent due to the possibility of creating a chaotic time series (ARCH, GARCH, TARCH, EGARCH, FIGARCH, CGARCH, etc.) [5];

4. Forecasting, which involves achieving the ultimate goal of time series analysis – forecasting [6].

Another important question which should be considered concerns choosing a proper computer program for time series analysis and forecasting.

The article deals with some problems of the aforementioned stages of time series analysis. As an example, the authors propose to consider modeling and forecasting the population of Ukraine by time series. The population data are presented as a univariate nonstationary time series. The next section of the article gives a review of the literature on the issue under study.

Currently, Ukraine is experiencing demographic decline. According to the State Statistics Service, during a ten month period of 2018, the population of Ukraine decreased by 166.3 thousand people [7]. This indicator is 18 % higher than that for the same period of 2017. The situation may be related to the migration outflow of the population and its natural decline. In view of the critical demographic situation, the authors propose using time series analysis to describe the population dynamics, for relevant authorities to promptly respond to challenges in this sphere.

Many modern scientists study demographic issues. The problem of the decrease in the working population (mostly through migration) and different ways to solve it are covered in articles [8], [9], and [10]. Statistical approaches to the analysis and forecasting of demographic data are also considered in the literature. For example, [11] presents new APC (age, period and cohort effects) models, methods, and empirical applications which are based on [1 – 6] fundamental works. Article [12] presents approaches of time series analysis for understanding the demographics of users of online social networks. The article highlights the digital aspect of demographic issues: social networks occupy an important place in the modern world of digital technologies and apply time series as a modeling and forecasting tool. Models and methods of [12] are also based on works [1 – 6].

There are a great number of computer programs for implementing analysis of time series. Their detailed consideration is presented in early fundamental works [1 – 6] and in the modern ones published in the heyday of software [8 – 12]. At first glance, the market for time series software does not seem to be underdeveloped, but with a more detailed analysis it becomes clear that there is definitely not enough available and high-quality software for analyzing time series. Professional software requires large financial expenditures. Professional statistical and mathematical software programs (SAS, Statistica, MATLAB) were used in articles [8 – 10], with MATLAB being employed even in earlier works [1 – 4]. Among free software packages, GRETL, TISEAN, and R (used in [11], [12]) are worth mentioning. Thus, it can be concluded that the distinguished leaders among the software used in this field are MATLAB, R.

MATLAB Econometrics Toolbox includes univariate Bayesian linear regression, univariate ARIMAX/GARCH composite models with several GARCH variants, multivariate VARX models and cointegration analysis for time series modeling and analysis. Considering the above mentioned, the authors chose MATLAB as the best program to fit the purpose of the study.

The next part of the article presents a step-by-step building of a time series model of Ukraine's population using MATLAB Econometrics Toolbox.

The statistics on the population of Ukraine are available at http://www.ereport.ru. The data are stored in the "population" array. The measurements are made at one year intervals which are stored in the "years" array. The statistics were used by the authors to develop a time series model. The MATLAB code for plotting the initial data is given below and the result of running the program segment is shown in Fig. 1.

```
clc
clear all
Population = [51.7; 51.9; 51.7; 51.3; 50.9;
50.4; 50.0; 49.5; 49.2; 48.8; 48.4; 48.1; 47.7;
47.4; 46.7; 46.3; 46.0; 45.7; 45.4; 45.1; 44.9;
44.6; 44.3; 44.4; 44.2];
years = [1992:1:2016]';
figure('Units', 'normalized', 'OuterPosition',
[0 0 1 1]);
title('Population of Ukraine, million peo-
ple');
plot(years, Population)
xlabel('years')
ylabel('the Population')
grid on
```

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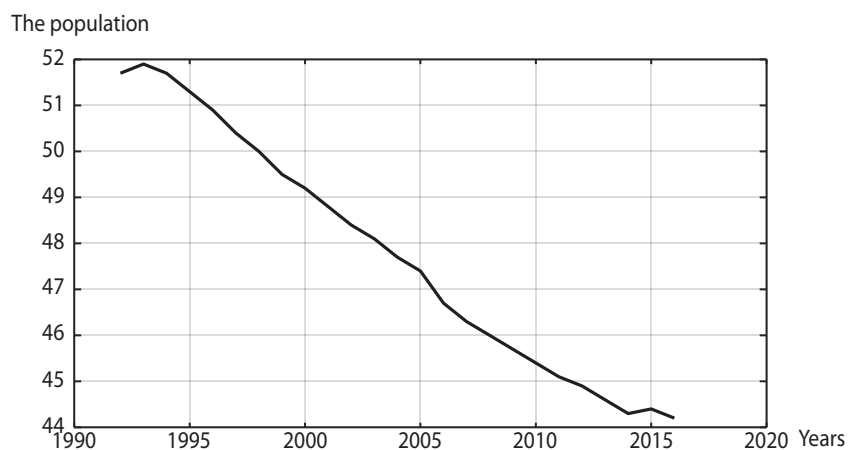
The population

Fig. 1. The population of Ukraine, million people

ow, let us create autocorrelation factor (ACF) and partial autocorrelation factor (PACF) plots to identify patterns in the above data. The idea is to identify the presence of AR and MA components in the residuals. The following is the MATLAB code to produce ACF and PACF plots. The result of the running of the program segment is shown in Fig. 2.

```
parcorr(Population)
autocorr(Population)
```

According to Fig. 2, b, autocorrelation coefficients are **more than 1**. It seems that in the MATLAB **parcorr** function gives wrong results. The phenomenon can be explained. This is not a bug but rather a result of different algorithms used to compute the PACF. For reference, the PACF is computed by fitting successive order AR models by OLS, retaining the last coefficient of each regression.

This is an approximation of the Yule-Walker equations' solution, but it should be suitable for reasonable sample sizes. These data are short (only 30 observations) and
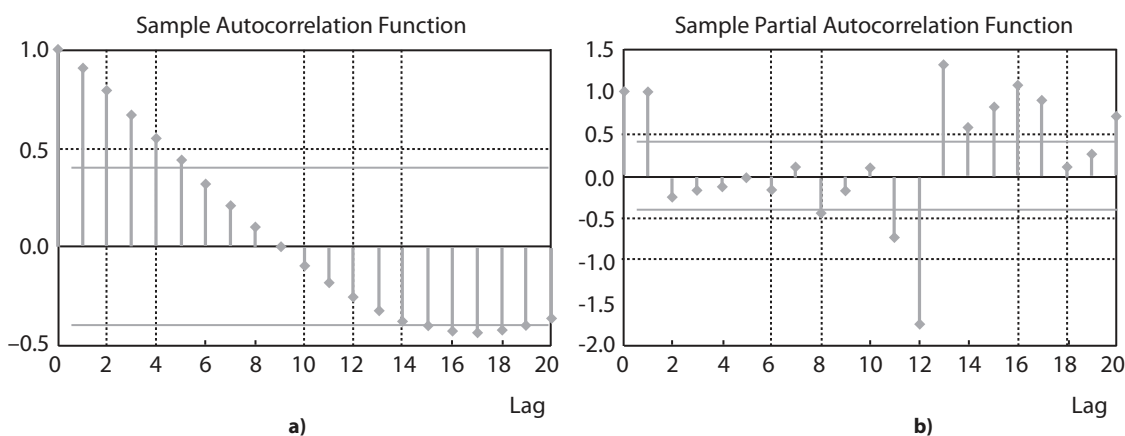
Fig. 2. Visualization of the ACF function (a) and PACF function (b)

they do not satisfy the assumptions that the Yule-Walker equations are based on. The authors recommend to use the following code in the situations:

```
if (par1 == 'Default') {
        par1 = 10*log10(length(x))
}
else {
        par1 <- as.numeric(par1)
}
        par2 <- as.numeric(par2)
        par3 <- as.numeric(par3)
        par4 <- as.numeric(par4)
        par5 <- as.numeric(par5)
if (par6 == 'White Noise') par6 <- 'white' else
```

```
par6 <- 'ma'
        par7 <- as.numeric(par7)
if (par8 != '') par8 <- as.numeric(par8)
        x <- na.omit(x)
        ox <- x
if (par8 == '') {
if (par2 == 0) {
      x <- log(x)
}
else {
      x <- (x ^ par2 - 1) / par2
}
}
else {
      x <- log(x,base=par8)
}
```

```
if (par3 > 0) x <- diff(x,lag=1,difference=par3)
if (par4 > 0) x <- diff(x,lag=par5,difference=
par4)
```

The result of the running of the program segment is shown in Fig. 3.

According to Fig. 3, a, the autocorrelation plot shows that the time series is not random but rather has a high degree of autocorrelation between adjacent and near-adjacent observations. Also, as seen from Figure 3, b, the partial autocorrelation plot demonstrates a clear
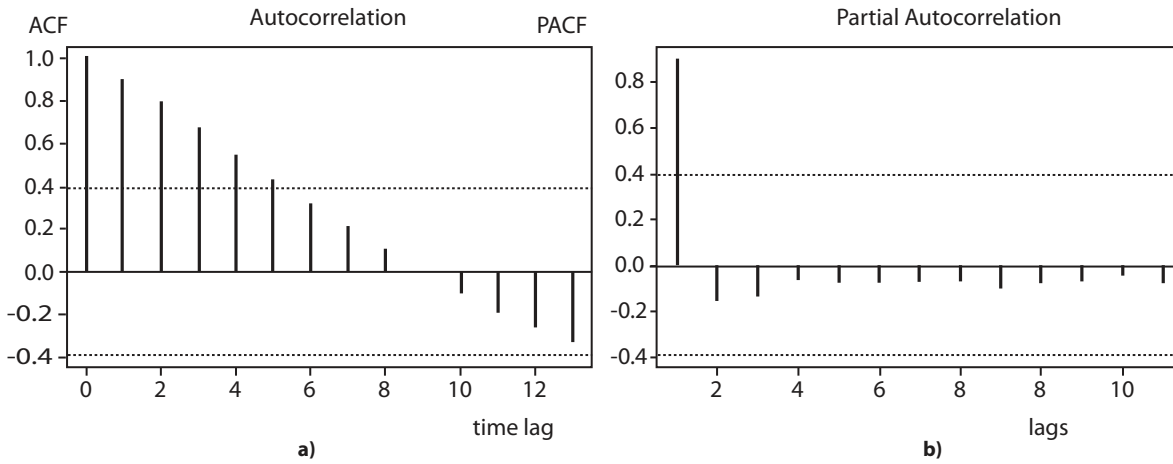

Fig. 3. Visualization of the ACF function (a) and PACF function (b) after using the authors' program

statistical significance for lags 1 and 2. The next few lags are in the middle of statistical significance.

We considered the ACF and PACF values up to lag 12, i.e., one-third of the data and 95 % confidence level. From these plots we can determine the type and order of the adequate model required to fit the series. Moreover, as the ACF values damp out rapidly for increasing lags, we can assume that the data series is stationary. The ACF and PACF plots indicate that an ARIMA model is appropriate.

The built-in ARIMA(p, D, q) function creates model objects for a stationary or unit root nonstationary linear time series model [13]. This includes moving average (MA), autoregressive (AR), mixed autoregressive and moving average (ARMA), integrated (ARIMA), multiplicative seasonal, and linear time series models which include a regression component (ARIMAX).

The input arguments of the built-in ARIMA function are shown in Table 1. The mathematical formulation of the ARIMA(p, D, q) model using lag polynomials is given below:

$$\phi(L)(1-L)^D y_t = \theta(L)\varepsilon_t, \text{ i.e.} \left(1-\sum_{i=1}^{p}\phi_i L^i\right)(1-L)^D y_t =$$
$$= \left(1+\sum_{j=1}^{q}\theta_j L^j\right)\varepsilon_t \quad (1)$$

Since the series used in this article are already transformed to log-returns, any further differentiation is not needed, and thus we may set d = 0. In this case the ARIMA(p, 0, q) turns into the ARMA(p, q) model. ARIMA has also a more complicated variant which allows to capture seasonality. Though it is available, we will not use it here (demographical data are nonseasonal). Mathematically the ARMA(p, q) model can be represented as

Table 1

Name of parameters	Description
P	Positive integer indicating the degree of the nonseasonal autoregressive polynomial
D	Nonnegative integer indicating the degree of nonseasonal integration in the linear time series
q	Positive integer indicating the degree of the nonseasonal moving average polynomial

$$\phi(L)y_t = \theta(L)\varepsilon_t, \text{ i.e.} \left(1-\sum_{i=1}^{p}\phi_i L^i\right)y_t =$$
$$= \left(1+\sum_{j=1}^{q}\theta_j L^j\right)\varepsilon_t \quad (2)$$

Since the MATLAB built-in function allows to estimate ARMA, from a technical point of view, it will be estimated further as the ARIMA(p, 0, q) model. But since there is no need to differentiate the dependent variable, this will be ARMA (p, q) specifications. The next MATLAB program code identifies the best fit ARIMA(p, 0, q) model. The following is the code for the same:

```
clc
clear all
r = [51.7; 51.9; 51.7; 51.3; 50.9; 50.4; 50.0;
49.5;
          49.2; 48.8; 48.4; 48.1; 47.7;
47.4; 46.7; 46.3;
          46.0; 45.7; 45.4; 45.1; 44.9;
44.6; 44.3; 44.4; 44.2];
max_p=5; max_q=1;
crit=zeros(max_p,max_q);
```

```
c=0;                                                          end
N=size(r,1);                                          crit(i,j)=bic; % <- change this line
for i=1:max_p                                     for AIC
   for j=1:max_q                                     end
      Mdl = arima(i,0,j);                        end
      [EstMdl,EstParamCov,logL,info]    =    fprintf('\nEstimates of BIC criterion: \n');
estimate(Mdl,r);                                  crit
      [aic,bic]=aicbic(logL,i+j,N);          fprintf('\n');
               if ((i==1) && (j==1)) ||    Mdl = arima(p,0,q);
(bic<c)                                                [EstMdl,EstParamCov,logL,info]         =
                    c=bic;                              estimate(Mdl,r);
                    p=i;                                [res,v] = infer(EstMdl,r);
                    q=j;
```

A step-by-step run of the program code is presented below:

```
ARIMA(1,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian
                            Standard            t
  Parameter      Value        Error        Statistic
  ----------   ----------   ------------   ----------
  Constant     -0.247391     0.837429      -0.295417
     AR{1}             1     0.0166123       60.1965
     MA{1}      0.466445      0.268057        1.7401
  Variance     0.034112     0.00917661       3.71727

ARIMA(2,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian
                            Standard            t
  Parameter      Value        Error        Statistic
  ----------   ----------   ------------   ----------
  Constant      0.656432     0.186673        3.51648
     AR{1}       1.77445     0.0522129       33.9849
     AR{2}     -0.789709     0.0537842      -14.6829
     MA{1}            -1      0.200678       -4.98312
  Variance     0.0130313    0.00593187       2.19682

ARIMA(3,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian
                            Standard            t
  Parameter      Value        Error        Statistic
  ----------   ----------   ------------   ----------
  Constant      0.531783     0.389661        1.36473
     AR{1}       1.96595      0.29192        6.73454
     AR{2}      -1.14885       0.5593       -2.05409
     AR{3}      0.170423     0.276867        0.61554
     MA{1}     -0.942271      0.35053       -2.68813
  Variance     0.0158196    0.00859024       1.84157

ARIMA(4,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian
                            Standard            t
  Parameter      Value        Error        Statistic
  ----------   ----------   ------------   ----------
  Constant       2.19704      1.31898        1.66571
     AR{1}      0.513897     0.233049         2.2051
     AR{2}      0.963821      0.31734        3.03719
     AR{3}     0.0220676     0.215029       0.102626
     AR{4}     -0.548489      0.20729       -2.64599
     MA{1}       0.69526     0.316827        2.19445
  Variance     0.0162225    0.00491912       3.29785

ARIMA(5,0,1) Model:
--------------------
Conditional Probability Distribution: Gaussian
                            Standard            t
  Parameter      Value        Error        Statistic
  ----------   ----------   ------------   ----------
  Constant       2.31268      1.36453        1.69485
     AR{1}      0.471412     0.401658        1.17366
     AR{2}      0.970034     0.322469        3.00815
     AR{3}     0.0838277      0.40442       0.207279
     AR{4}      -0.52745     0.314787       -1.67558
```

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```
     AR{5}      -0.0489605      0.361783       -0.135331
     MA{1}       0.711955       0.4246          1.67677
  Variance       0.0161973      0.004998        3.24076

Estimates of BIC criterion:
crit =
   -7.0680
  -28.8448
  -24.1408
  -15.9924
  -12.8126

   ARIMA(2,0,1) Model:
   -------------------
  Conditional Probability Distribution: Gaussian
                              Standard          t
   Parameter       Value       Error       Statistic
   ----------    ----------   ----------   ----------
   Constant       0.656432     0.186673      3.51648
     AR{1}        1.77445      0.0522129    33.9849
     AR{2}       -0.789709     0.0537842   -14.6829
     MA{1}           -1        0.200678     -4.98312
  Variance        0.0130313    0.00593187    2.19682
```

Choosing the best ARIMA model is based on Akaike or Bayesian information criteria. The best fit model selected is ARIMA(2,0,1). It means that the degree of the nonseasonal autoregressive polynomial is two, the degree of nonseasonal integration in the linear time series is zero, the degree of the nonseasonal moving average polynomial is one. Mathematically, the resulting model can be presented as follows:

$$\phi(L)y_t = \theta(L)\varepsilon_t, i.e. (1-\phi_1 L^1 - \phi_2 L^2)y_t = = (1+\theta_1 L^1)\varepsilon_t \qquad (3)$$

We are going to apply the ARIMA model with well-chosen parameters for time-series forecasting. This process includes two stages: estimating and forecasting. The following is the MATLAB code to estimate ARIMA model parameters using initial values and to forecast the population of Ukraine for the next 3 years (2017, 2018, and 2019) applying the above mentioned model. Since the forecast includes the year of 2016, it allows us to compare the past data with the reproduced forecast results.

```
   ARIMA(2,0,1) Model:
   -------------------
  Conditional Probability Distribution: Gaussian
                              Standard          t
   Parameter       Value       Error       Statistic
   ----------    ----------   ----------   ----------
   Constant       0.656432     0.186673      3.51648
     AR{1}        1.77445      0.0522129    33.9849
     AR{2}       -0.789709     0.0537842   -14.6829
     MA{1}           -1        0.200678     -4.98312
  Variance        0.0130313    0.00593187    2.19682

PopulationFit =
   ARIMA(2,0,1) Model:
   -------------------
  Distribution: Name = 'Gaussian'
            P: 2
            D: 0
            Q: 1
     Constant: 0.656432
          AR: {1.77445 -0.789709} at Lags [1 2]
         SAR: {}
          MA: {-1} at Lags [1]
         SMA: {}
     Variance: 0.0130313
```

```
PopulationModel = arima(2,0,1)
PopulationFit = estimate(PopulationModel,Popu
lation)
[Y,YMSE] = forecast(PopulationFit,4,'Y0',Popu
lation)
```

The result of the program fragment is shown below.

```
PopulationModel =
   ARIMA(2,0,1) Model:
   -------------------
  Distribution: Name = 'Gaussian'
            P: 2
            D: 0
            Q: 1
     Constant: NaN
          AR: {NaN NaN} at Lags [1 2]
         SAR: {}
          MA: {NaN} at Lags [1]
         SMA: {}
     Variance: NaN
```

```
Y =
   44.2045
   44.2106
   44.0516
   43.9528

YMSE =
    0.0403
    0.2139
    0.8230
    2.5207
```

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The graph of the generated forecast is presented in Fig. 4.

The line plot shows the observed values compared to the rolling forecast estimates. In general, our forecasts align with the true value (the year of 2016) very well, showing a downward trend which started in the year of 2016 and finished in 2019.
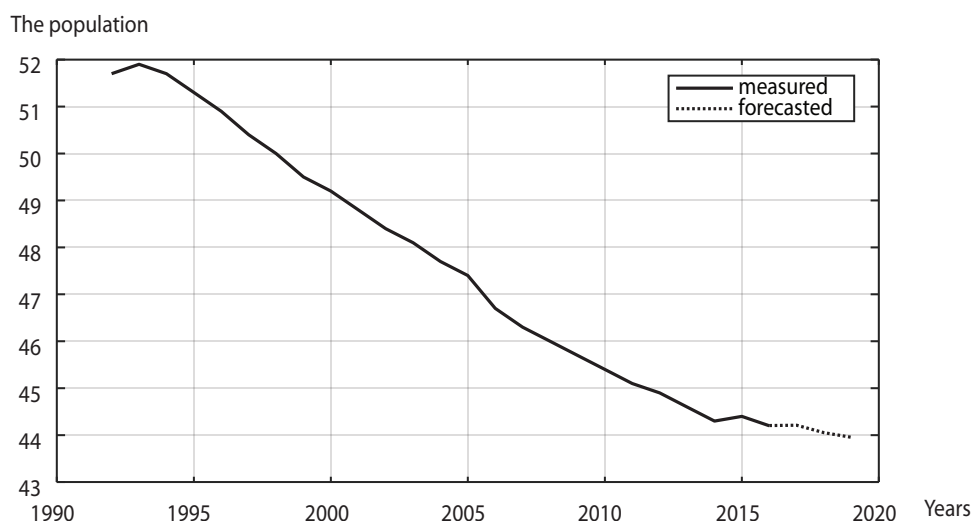
Particular attention should be paid to quality of the forecast. The automatically calculated mean-square error values (YMSE array) are close to zero, which indicates a good fit of the selected model.

Unfortunately, the forecast trend shows that there may be a shortage of skilled labor in Ukraine within 10 years.

Fig. 4. The forecast of time series data using the ARIMA model

CONCLUSIONS

In the course of the research, different methods of and approaches to time series analysis were considered. Unfortunately, a universal approach for addressing problems in this field has not been found yet. The methods developed in the 60s of the last century (and some at the beginning of the 19th century) are still popular along with the MATLAB Econometrics Toolbox which is considered in the article. This is partly due to the fact that the task of forecasting, like any other task arising in the process of working with data, is in many ways a creative and certainly research. Despite a great number of formal quality metrics and methods for estimating parameters, for each time series it is often necessary to select and try something different: e.g., you can get correlation coefficients greater than 1 or thoughtlessly choose the parameters of the ARIMA model.

To find balance between quality and labor costs is also of importance. The MATLAB Econometrics Toolbox demonstrates outstanding results with proper tuning, may require more than one hour of manipulations with the programming and additional settings, while a simple linear regression can be built in 10 minutes with more or less comparable results.

The time series of the population of Ukraine were modeled as one-dimensional time series with nonstationary data using the ARIMA model. Thus, the authors recommend to apply the approaches and methods used in the article, as well as the developed algorithms only for time series of the kind. ∎

LITERATURE

1. Cohen J., Cohen P., West S. G., Aiken L. S. Applied multiple regression/correlation analysis for the behavioral sciences (3rd ed.). Mahwah, NJ : Lawrence Erlbaum, 2003.

2. Broughton S. A., Bryan K. Discrete Fourier Analysis and Wavelets: Applications to Signal and Image Processing. New York : Wiley, 2008. 72 p.

3. Murphy K. Machine Learning: A Probabilistic Perspective. MIT, 2012. 16 p.

4. Everitt B. The Cambridge Dictionary of Statistics. Cambridge, UK; New York : Cambridge University Press, 1998.

5. Muhammad Imdad Ullah. Basic Statistics and Data Analysis. Retrieved 2 January 2014. URL: http://itfeature.com/

6. Mormann F., Andrzejak R. G., Elger Ch. E., Lehnertz K. Seizure prediction: the long and winding road. *Brain*. 2007. No. 130 (2). P. 314–333.

DOI: 10.1093/brain/awl241.PMID17008335

7. Олексенко Р. И., Афанасьева Л. В. Экономико-правовой механизм реализации новой модели современной культурной политики Украины // Актуальные проблемы экономики, менеджмента и маркетинга в современных условиях : материалы Междунар. науч.-практ. заочной конф. Мелитополь, 2019. С. 421–425.

8. Wessel T., Turner L. M., Nordvik V. Population dynamics and ethnic geographies in Oslo: the impact of migration and natural demographic change on ethnic composition and segregation. *Journal of Housing and the Built Environment*. 2018. Vol. 33. No. 4. P. 789–805.

9. Koons D. N. et al. A life history perspective on the demographic drivers of structured population dynamics in changing environments. *Ecology letters*. 2016. Vol. 19. No. 9. P. 1023–1031.

10. Koons D. N., Arnold T. W., Schaub M. Understanding the demographic drivers of realized population growth rates. *Ecological Applications*. 2017. Vol. 27. No. 7. P. 2102–2115.

11. Yang Y., Land K. C. Age-period-cohort analysis: New models, methods, and empirical applications. Chapman and Hall/CRC, 2016.

12. Culotta A., Kumar N. R., Cutler J. Predicting the Demographics of Twitter Users from Website Traffic Data // AAAI. 2015. P. 72–78.

13. Shapour Mohammadi, Hossein Abbasi-Nejad. A Matlab Code for Univariate Time Series Forecasting // Computer Programs 0505001, University Library of Munich, Germany, 2005. URL: https://ideas.repec.org/c/wpa/wuwppr/0505001.html

REFERENCES

Broughton, S. A., and Bryan, K. *Discrete Fourier Analysis and Wavelets: Applications to Signal and Image Processing*. New York: Wiley, 2008.

Cohen, J. et al. *Applied multiple regression/correlation analysis for the behavioral sciences*. Mahwah, NJ: Lawrence Erlbaum, 2003.

Culotta, A., Kumar, N. R., and Cutler, J. "Predicting the Demographics of Twitter Users from Website Traffic Data". In *AAAI*, 72-78, 2015.

Everitt, B. *The Cambridge Dictionary of Statistics*. Cambridge, UK; New York: Cambridge University Press, 1998.

Koons, D. N. et al. "A life history perspective on the demographic drivers of structured population dynamics in changing environments". *Ecology letters*, vol. 19, no. 9 (2016): 1023-1031.

Koons, D. N., Arnold, T. W., and Schaub, M. "Understanding the demographic drivers of realized population growth rates". *Ecological Applications*, vol. 27, no. 7 (2017): 2102-2115.

"Muhammad Imdad Ullah. Basic Statistics and Data Analysis. Retrieved 2 January 2014". http://itfeature.com/

Mormann, F. et al. "Seizure prediction: the long and winding road". *Brain*, no. 130(2) (2007): 314-333. DOI: 10.1093/brain/awl241. PMID17008335

Murphy, K. *Machine Learning: A Probabilistic Perspective*. MIT, 2012.

Oleksenko, R. I., and Afanaseva, L. V. "Ekonomiko-pravovoy mekhanizm realizatsii novoy modeli sovremennoy kulturnoy politiki Ukrainy" [Economic and legal mechanism for the implementation of a new model of modern cultural policy of Ukraine]. *Aktualnyye problemy ekonomiki, menedzhmenta i marketinga v sovremennykh usloviyakh*. Melitopol, 2019. 421-425.

Shapour, Mohammadi, and Hossein, Abbasi-Nejad. "A Matlab Code for Univariate Time Series Forecasting". Computer Programs 0505001, University Library of Munich, Germany, 2005. https://ideas.repec.org/c/wpa/wuwppr/0505001.html

Wessel, T., Turner, L. M., and Nordvik, V. "Population dynamics and ethnic geographies in Oslo: the impact of migration and natural demographic change on ethnic composition and segregation". *Journal of Housing and the Built Environment*, vol. 33, no. 4 (2018): 789-805.

Yang, Y., and Land, K. C. *Age-period-cohort analysis: New models, methods, and empirical applications*. Chapman and Hall/CRC, 2016.

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